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A PHOTOMETRIC STUDY OF EI CEPHEI

BY

ⓒ LORENCE TOMASZEWSKI

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "A PHOTOMETRIC STUDY OF EI CEPHEI" submitted by LORENCE TOMASZEWSKI in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

A photometric re-examination of the eclipsing variable EI CEPHEI was undertaken using the 12.5 inch Cassegrain telescope housed at the Devon observatory. The intensity measurements were made in four colours defined by the uvby filter system. The method of analysis employed to solve for the radii, luminosities, and angle of inclination of the orbit was the procedure suggested by M. Kitamura in 1965. A FORTRAN program, EVAL, was written to calculate the various parameters required to use the Kitamura method. Observations of the variable were made from August, 1972 to October, 1973 from which 208 data points in each colour were obtained. The data when plotted to form the light curves demonstrated a high degree of scatter. Although it was possible to easily observe the minima, the light curves were of insufficient accuracy to yeild a final solution for the orbital elements. It was, however, possible to show that the primary minimum is due to an occultation (eclipse of the smaller star by the larger component) while the secondary minimum is due to a transit (eclipse of the larger star).

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TABLE OF CONTENTS

CHAPTER 1 PAGE

INTRODUCTION

1.1 Binary Stars........................... 1

1.2 Photometry............................. 2

1.3 The Classifications and Astronomical Importance of Eclipsing Variables........ 6

1.4 Statement of the Problem................. 9

CHAPTER 2

THEORETICAL TREATMENT OF ECLIPSING VARIABLES

2.1 Geometry of the Eclipses................ 14

2.2 Theoretical Light Curves................ 18

2.3 The Analysis of the Light Curves......... 22

(i) Russell's Method................. 24

(ii) Kopal's Method.................. 25

2.4 Method of Kitamura...................... 27

CHAPTER 3

OBSERVATIONAL EQUIPMENT AND EXPERIMENTAL TECHNIQUE

3.1 Observing Equipment

(i) Telescope......................... 33

(ii) Photometer........................ 33

(a) Photomultiplier............. 34

(iii) Filters........................... 34

(iv) Miscellaneous.......................... 36

3.2 Systematics.............................. 37

(i) Extinction.......................... 38

(ii) Transformation...................... 40

3.2b Differential Photometry.................. 42

3.2c Time..................................... 44

(i) Light Time........................... 45

3.3a Observational Techniques................. 46

3.3b Reduction of Observational Data.......... 47

CHAPTER 4

THE OBSERVATIONS

4.1 The Observation of EI CEP and Comparison Star.. 48

(i) Light Curves........................ 60

(ii) The Individual Minima.............. 69

(iii) The Colour Indices................. 70

4.2 The Observations of the Comparison Star and Check Star.................................. 70

4.3 The Observations of Standard Stars....... 80

CHAPTER 5

ANALYSIS OF THE LIGHT-CURVES

5.1 The Computer Program EVAL................ 82

5.2 The Results Obtained by use of EVAL...... 85

5.3 Conclusions Concerning EI CEP............. 85

5.4 Concluding Remarks.........................102

APPENDIX 1. A listing of and Operating Instructions for the Computer program CORR...................108

APPENDIX 2. A listing of and Operating Instructions for the Computer program EVAL...................115

REFERENCES....................................105

VITA...128

LIST OF TABLES

TABLE	DESCRIPTION	PAGE
1.1	Spectroscopic and Photometric Elements	12
4.1	Observations Obtained on EI CEPHEI	49
4.2	The Means and Standard Deviations of the Colour Differences Corresponding to Various Sections of the Light Curves	74
4.3	Check Star Data	75
5.1	Computer Results for the u-Filter	86
5.2	Computer Results for the v-Filter	87
5.3	Computer Results for the b-Filter	88
5.4	Computer Results for the y-Filter	89
5.5	Depth of Minima	90
5.6	Normalized Values of the Luminosity	91

LIST OF FIGURES

FIGURE		PAGE
2.1	Geometry of the Eclipses	16
2.2	Schematic Representation of Limb-Darkening	19
4.1	Light-Curve of EI CEP u-Filter	56
4.2	Light-Curve of EI CEP v-Filter	57
4.3	Light-Curve of EI CEP b-Filter	58
4.4	Light-Curve of EI CEP y-Filter	59
4.5	Primary Minimum u-Filter	61
4.6	Secondary Minimum u-Filter	62
4.7	Primary Minimum v-Filter	63
4.8	Secondary Minimum v-Filter	64
4.9	Primary Minimum b-Filter	65
4.10	Secondary Minimum b-Filter	66
4.11	Primary Minimum y-Filter	67
4.12	Secondary Minimum y-Filter	68
4.13	EI CEPHEI Colour Index b minus y	71
4.14	EI CEPHEI Colour Difference m_1	72
4.15	EI CEPHEI Colour Difference c_1	73
4.16	Check Star Minus Comparison Star u-Filter	76
4.17	Check Star Minus Comparison Star v-Filter	77
4.18	Check Star Minus Comparison Star b-Filter	78

FIGURE PAGE

4.19 Check Star Minus Comparison Star y-Filter 79

5.1 EI CEPHEI Normalized Light-Curve u-Filter 97

5.2 EI CEPHEI Normalized Light-Curve v-Filter 98

5.3 EI CEPHEI Normalized Light-Curve b-Filter 99

5.4 EI CEPHEI Normalized Light-Curve y-Filter 100

CHAPTER 1

INTRODUCTION

1.1 BINARY STARS

With the development of the telescope it was found that certain stars formerly observed to be singular could be resolved into components. Such stars were considered as little more than astronomical curiosities since it was presumed they were merely stars that possessed similar spatial coordinates. The attitude towards these objects changed however with the announcement by Halley in 1718 that some of the brighter stars, Sirius, Arcturus, and Aldebaran were in motion. In 1767, John Michell proposed an argument based on the theory of probabilities that some of these double stars could be systems joined together under some general law. The real beginning to double star astronomy however is credited to Sir William Herschel, who carefully investigated such objects throughout the last quarter of the eighteenth century. The culmination of his work was presented in a paper written 1803(1) in which he proved by actual demonstration from measurements he had made that certain double stars were true binary systems. Stars where it is possible to optically resolve the individual components are designated as visual binaries.

Another class of binaries was discovered with the aid of the spectroscope. Stars that still appeared single in a telescope were found to display periodic oscillations of spectral lines about their mean positions. This movement of the spectral lines of the so-called spectroscopic binaries was attributed to the Doppler effect resulting from the motions of the stars in their mutual orbits. If the plane of the orbit of one of these spectroscopic binaries should happen to fall close to the line of sight from the earth then it is possible that the two components will periodically eclipse one another. During the passage of one star in front of its companion, the apparent brightness of the system as a whole will diminish. Such a celestial object is classified as the eclipsing binary or eclipsing variable. A common point among binaries is that by proper mathematical analysis, pertinent information about the star system can be obtained. This is especially true for the eclipsing binary and therefore it has been chosen as the basis of this thesis.

1.2 PHOTOMETRY.

The distinguishing feature of eclipsing binaries is their light variations. Therefore, it is essential as a method of detection to measure any fluctuations in light

intensity. This problem can be resolved by employing the general concepts of photometry; astronomical photometry is concerned with measurement of stellar brightness (magnitudes). The first device used as a photometer was the eye. Stars were compared to chosen standards in an attempt to catalogue them according to their apparent brightness. This method proved extremely inaccurate due to the eye's inability to accurately evaluate intensity intervals. To aid the eye in making more accurate estimates the use of null-methods was instituted. One such device applying this technique was the polarizing photometer.(2) Here a telescopic image of the real star was compared to the image of an artificial star whose brightness could be varied precisely by a rotating polarizing prism. Although it was possible to obtain stellar magnitudes with uncertainties as low as 0.04 magnitude, there still existed large systematic errors due to differences in color sensitivity between different observers.

With the development of the photographic plate in 1850 a new system of measurement became possible. The fundamental principle of photographic photometry is that equal intensities produce equal photographic effects under identical conditions.

These conditions included the emulsion, the exposure time, temperature and humidity during exposure, and the size and structure of the optical image. Because of the difficulties in controlling changes in some of these conditions and the non-linear response of the emulsion with exposure time, photographs of the brightest and faintest stars are often incorrectly exposed. Despite these drawbacks, photographic photometry has the unique capacity to simultaneously record large numbers of objects in a limited field which is indispensible in stellar statistics, variable star surveys and investigations of rich star clusters. By establishing reliable magnitude sequences in the region under study it became possible to reduce the errors in estimation to 0.02 magnitude.

The most recent advance, and that used throughout this project, has been the development of the photoelectric photometer an electronic device that displays a linear response over a broad range of stellar magnitude. The basis for such an instrument is the photoelectric effect discovered by Hallwachs in 1888. Hallwachs observed that when light fell on certain substances, a measurable electric current proportional to the incident light intensity resulted and, therefore, an accurate measure of this current could

produce an accurate determination of the light intensity. Early work in this field of photometry was carried out independently by J. Stebbins in the U.S. and P. Guthnik in Germany. The first photocells used had a selenium photocathode of low sensitivity which restricted work to the very bright stars. After 1911, more sensitive potassium hydride cells were constructed. Despite the development of these more sensitive cells, the measurement of the output current still required the use of elaborate electrometers which consumed valuable quantities of man-hours in the course of measuring a single light-curve. With the improvement of electronic amplifiers, it became possible to increase feeble cell currents to the point where ordinary galvanometers could be used. A major improvement occurred with the production of the RCA 1P21 photoelectric cell employing the use of secondary emission which coupled a high internal gain with a broad linear response. This new cell together with new recording instruments resulted in accuracies of 0.01 magnitude. The limiting effects on the accuracy of photoelectric measurements are imposed by the atmosphere (e.g. changes in transparency and scintillation effects), and noise in the photomultiplier which will limit the magnitudes that can be measured.

1.3 THE CLASSIFICATIONS AND ASTRONOMICAL IMPORTANCE OF ECLIPSING VARIABLES

By using the photoelectric photometer and its ancillary equipment, it is possible to obtain highly accurate observational data. These data for an eclipsing binary consists of a relation between apparent brightness and time. A graph of these measured quantities is called a light-curve. A study of the many light-curves obtained to date has shown that it is possible to classify the eclipsing binaries as follows:

(1) no evidence of any significant extra-eclipse variation, therefore the light between eclipses can be regarded as constant.

(2) the light is found to vary due to reflection and ellipticity effects but the moments at which the eclipse begins and ends are still distinguishable.

(3) the changes in light between eclipses are comparable to those caused by the eclipses and make it impossible to approximate the beginning and end of an eclipse.

These three classifications which are based on the degree of proximity effects correspond approximately to the following models which are named after the prototype star:

(1) Algol: component stars well-separated, so stars are essentially spherical; orbit is very close to circular.

(2) β Lyrae: components so close as to be virtually in contact, tidal effects distort stars into elongated ellipsoids, eccentric orbits.

(3) W Ursae Majoris: the components are physically united and as such act as a singular unit, period of revolution is less than 12 hours due to compactness of system.

These classifications, corresponding to theoretical models are important in the analysis of all light curves. It is possible through techniques to be discussed in the next chapter to solve for:

(1) the radii of the two components r_1, r_2 in terms of mean distance, $a = (a_1 + a_2)$

(2) the luminosities L_1, L_2 in terms of the total luminosity of the system outside eclipse.

(3) the angle of inclination of the orbit, i to the line of sight of the observer. Angle i approaches 90° as the line of sight approaches plane of orbit.

(4) the ratio of the surface brightness

$$\frac{J_1}{J_2} = \frac{L_1}{L_2} \frac{r_2^2}{r_1^2}$$

If the eclipsing binary is also studied spectroscopically and if both spectra are visible, then an analysis of the velocity curve given by the Doppler shift of the spectral lines will yield the quantities a_1 sin i, a_2 sin i and m_1 $\sin^3 i$, m_2 $\sin^3 i$. Therefore a combination of photometric and spectroscopic data will yield the absolute values of the radii, masses, and densities of the eclipsing system. When the problem of an eclipsing binary system is studied astrophysically more information than merely the dimensions or the surface characteristics of the components or the geometry of the orbits are provided. With a single star it is impossible to probe the gravitational field or learn any thing about the surface brightness. However, place a companion in close proximity and properties of the combined gravitational field can be deduced from characteristics of the motion (e.g. apsidal advance) or by their mutual distortion resulting in changes in the light curve. The distribution of intensity over the apparent disks can be studied through the variation in brightness provoked by axial rotation of the distorted components. Non-radial oscillations of tidal origin in a close eccentric system can (through tidal lag) disclose the degree of viscosity of the outer layers of the component stars.

1.4 STATEMENT OF THE PROBLEM

The basic aims of this research project are listed below:

(1) Obtain complete photoelectric light curves in the uvby intermediate-band photometric system following the procedures of differential photometry.

(2) If possible, reduce the photometry to the standard uvby system.

(3) In carrying out (1) and (2) it will be necessary to determine extinction coefficients.

(4) In carrying out (1) and (2) it will be possible to formulate conclusions regarding the precision with which intermediate-band photometry can be done on faint stars with a small telescope and an unrefrigerated photomultiplier.

(5) Analyse the light curves for the orbital elements and combine these with published spectroscopic elements.

The eclipsing system under investigation is EI CEPHEI [BD + 75°079 = HD205234 = GC30132]. This star is a Henry Draper spectral type F0 with a visual magnitude of 7.67. It is of particular importance since EI CEP is one of only 19

known main sequence eclipsing variables of spectral classes B9-G for which accurate determination of the component masses should be possible. EI CEP was first established not as an eclipsing variable but as a double-line spectroscopic binary by J.S. Plaskett of the Dominion Astrophysical Observatory (Victoria) in 1921.(3) A provisional set of elements of the spectroscopic orbit was calculated in 1935 by W.E. Harper of the D.A.O.(4) The possibility that EI CEP was also a photometric binary was suggested by W. Strohmeier(5) at the Bamburg Observatory in 1958 during a search for new variables. A photometric study was undertaken by A. Abrami(6) of the Astronomical Observatory of Trieste during the period of 1959-1965 using an unfiltered photometer. Abrami's results confirmed Strohmeier's earlier suggestion that EI CEPHEI was indeed an eclipsing binary and he further calculated it had a period of 8.439365 days with two narrow minima of approximately the same duration and with depths of about 0.507 and 0.422 magnitude respectively. The analysis of Abrami's data for the orbital elements was hampered by the presence of third light constituting 9.0% of the total luminosity. The inclusion of this extraneous light brought with it a measurable uncertainty in the values of the photometric elements. In 1969 S. Kriz(7) of the

Czechoslovak Academy of Sciences used the combined photo metric and spectroscopic data for a comparison with theoretical models of main-sequence stars as proposed by Iben and Horn. Kriz concluded that the models agreed well with most of the observational data available except for EI CEPHEI which deviated in all comparisons. This deviation from the theoretical predictions was attributed by Kriz to the considerable uncertainty arising from the combination of Harper's preliminary spectroscopic data with Abrami's somewhat uncertain light elements. Kriz finally concluded that these uncertainties were large enough to warrant a new set of orbital elements. The first reinvestigation of EI CEP was carried out by D. Popper(8) in 1970 with a recalculation of the spectroscopic orbit from a new set of spectrograms. The objective of this thesis will therefore involve a multi-colour photometric study of EI CEPHEI in an attempt to obtain more accurate photometric elements.

TABLE 1.1

SPECTROSCOPIC ELEMENTS

	Harper (1935)	Popper (1970)
Period (days)	8.4455	8.439365
Eccentricity	0.039	
K_1 (km/sec)	79.71	81.2 $\mp$ 0.5
K_2 (km/sec)	80.28	76.9 $\mp$ 0.5
T_0	2417978.327	2436820.4665
a sin i (solar radii)	26.7	26.4 $\mp$ 0.1
$m_1 \sin^3 i$ (solar masses)	1.80	1.68 $\mp$ .003
$m_2 \sin^3 i$ (solar masses)	1.79	1.78 $\mp$ .003

PHOTOMETRIC ELEMENTS

	Abrami (1966)
k ($r_s \div r_g$)	0.774
Inclination, i (degrees)	87.97
r_s	0.087
r_g	0.113
L_a	0.385
L_b	0.522
Third light (L_3)	0.093

a_1 (km)	9.25×10^6
a_2 (km)	9.32×10^6
m_1 (solar masses)	1.79
m_2 (solar masses)	1.78
r_s (solar radii)	2.33
r_g (solar radii)	3.01

The quantities K_1 and K_2 represent the semi-amplitudes of the radial velocity changes versus the phase (position of the two components) for Star 1 and Star 2. The value denoted by the letter a is the apparent separation of the two stars expressed in solar radii. T_0 represents an accurately measured time of primary minimum. The values $r_s, r_g, L_a = L_s, L_b = L_g$ represent the radius and luminosity of the small and large component respectively normalized by a $(= a_1 + a_2)$ for the radii and the total luminosity outside eclipse for L_s and L_g. The angle of inclination is the angle formed by the line of sight of the observer and the normal to the plane of orbit of the system. The presence of third light indicates that an unseen object is contributing to the total output of the system.

CHAPTER 2

THEORETICAL TREATMENT OF ECLIPSING VARIABLES

2.1 GEOMETRY OF THE ECLIPSES

The interpretation of any eclipsing star system is based on a comparison of observations with a simple model or a series of increasingly more complex models. In the case of an eclipsing binary, the simplest system consists of two spherical stars of uniform brightness moving in circular orbits. Although this model is not entirely realistic it does provide a useful and often accurate starting point.

At the conjunctions of the stars there will be alternating eclipses, while out of eclipse the light output will remain constant. Because of the spherical nature of the stars and their circular orbits, the minima will be symmetrical, of equal duration but (generally) unequal depth, and will occur exactly one-half an orbital period apart. A careful study of the light curve will reveal either a continuous variation of light during both minima corresponding to partial eclipses or a phase of constant light at the bottom of one (or both) minimum due to alternating total and annular eclipses. In general any minimum, total or partial, arising from an eclipse of the smaller star by the larger component is termed an occultation whereas the reverse configuration is called a transit. Difficulties in

discriminating between the various types may arise if the eclipse is nearly grazing or if only one minimum has been detected. In studying a light curve where both eclipses have been observed, the deeper minimum is referred to as the primary and the shallower minimum as the secondary. The deeper minimum will always correspond to the eclipse of the star of greater surface brightness.

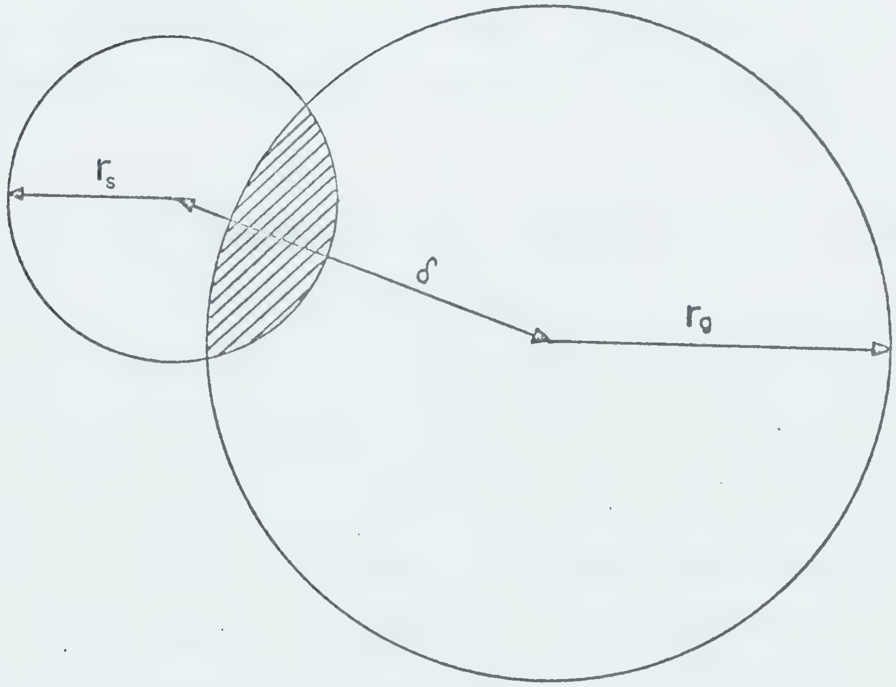
At any moment during an eclipse, one component eclipses a certain area of the other; however, during the corresponding phase of the alternate eclipse, the geometrical relation of the two disks remains unchanged and the other star eclipses an equal area (FIG. 2.1) but not necessarily an equal fraction of the total area of the other star. If, as we have assumed, both components are uniformly bright, i.e. the distribution of surface brightness is equal at each point on the stellar disk, then the changes in light during both minima should be similar and differ only on a scale corresponding to the ratio of the surface brightness of the two stars. This type of simple relation is complicated by the fact that most stars display some degree of limb darkening. This phenomenon was first noticed from observations of the sun. The limb (the outer-most part of the visible surface) of the sun was observed to be darker than the central region. The reasoning behind this effect is

FIG. 2.1

GEOMETRY OF THE ECLIPSES

The eclipse is an occultation if the eclipsed (shaded) area is on the smaller star.

The eclipse is a transit if the eclipsed area is on the larger star.



These conditions included the emulsion, the exposure time, temperature and humidity during exposure, and the size and structure of the optical image. Because of the difficulties in controlling changes in some of these conditions and the non-linear response of the emulsion with exposure time, photographs of the brightest and faintest stars are often incorrectly exposed. Despite these drawbacks, photographic photometry has the unique capacity to simultaneously record large numbers of objects in a limited field which is indispensible in stellar statistics, variable star surveys and investigations of rich star clusters. By establishing reliable magnitude sequences in the region under study it became possible to reduce the errors in estimation to 0.02 magnitude.

The most recent advance, and that used throughout this project, has been the development of the photoelectric photometer an electronic device that displays a linear response over a broad range of stellar magnitude. The basis for such an instrument is the photoelectric effect discovered by Hallwachs in 1888. Hallwachs observed that when light fell on certain substances, a measurable electric current proportional to the incident light intensity resulted and, therefore, an accurate measure of this current could

associated with the varying optical depths of the sun's outer surface. If we observe the sun near the centre we receive light from a layer corresponding to a certain depth and temperature. However, near the edge one can see a comparable distance but the deepest layer is higher than at the centre (see FIG. 2.2); therefore, the temperature is lower and the sun appears darker at that point. The net effect of limb-darkening on the light curve is to decrease the overall loss of light during an eclipse.

2.2 THEORETICAL LIGHT CHANGES

For our presumed model involving circular motion, the longitude of the orbit, θ, measured from primary minimum is given by

$$\theta = \frac{360^\circ}{P} (t - t_o)$$

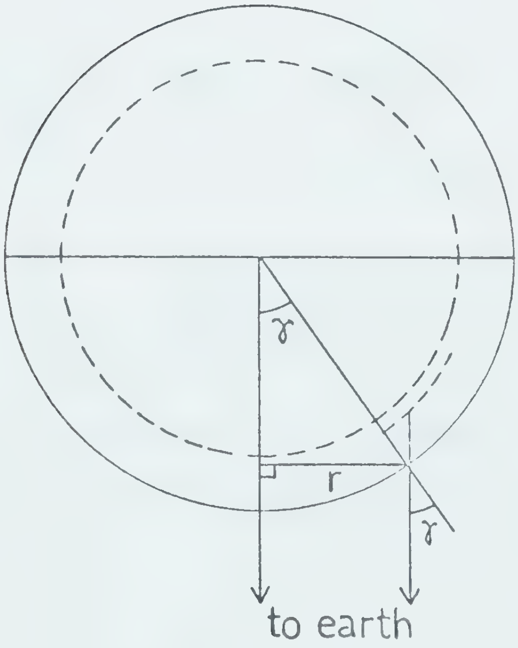
where P is the period (time between two successive primary minima) and t_o the epoch of primary conjunction both of which are expressed in Julian days. In order to relate the position as a function of time to the light change during an eclipse, we consider two stars with an apparent separation δ between their centres where from geometrical considerations:

$$\delta^2 = \cos^2 i + \sin^2 i \sin^2\theta \quad (2.1)$$

FIG. 2.2

SCHEMATIC EXPLANATION OF LIMB DARKENING

The light from the centre is emitted from a deeper region than at the limb. The angle formed by the intersection of the surface normal and the line of sight is referred to as the angle of foreshortening.



The fractional loss of light at any moment during an eclipse can be defined as a function depending upon r_a, the radius of the smaller component, r_b, the radius of the larger star, and δ, the separation of the centres. For the case of uniform brightness, the fractional loss of light, f, is the ratio of the eclipsed area to the total area of the eclipsed star, while for the completely limb-darkened case, f is the ratio of the light emitted from the eclipsed area to the total light of the star being eclipsed. The function f is homogeneous in the three elements r_a, r_b, and δ, it can therefore be made to depend on only two ratios formed between them. The most advantageous of these are the ratio of the radii, k and the geometrical depth of eclipse, p, where by definition:

$$k = \frac{r_a}{r_b} \qquad 0 \leq k \leq 1 \qquad (2.2)$$

and

$$p = \frac{\delta - r_b}{r_a} \quad , \qquad \begin{array}{ll} p = 1 & \text{moment of first contact} \\ p = -1 & \text{moment of internal tangency} \end{array}$$

The two radii are normalized to the unknown radius of the circular orbit. The loss of light during eclipse can be expressed as (9).

$$L = \int_s J \cos \gamma \, d\sigma \qquad (2.3)$$

where J is the surface brightness, γ the angle of foreshortening, $d\sigma$ the surface element, and s the region of integration. From the principles of radiative transfer in stellar atmospheres, the change in surface brightness across the disk of a star follows a cosine law of the form

$$J(\gamma) = J(o)(1-u+u \cos \gamma) \qquad (2.4)$$

where u is the coefficient of limb darkening and γ as before the angle of foreshortening. If the change in light as found from equation 2.3 is compared to the light emanating from the entire disk of the eclipsed star, the resulting value will represent the fractional loss of light where the f-functions in general are complex mathematical expressions involving transcendental functions. The fractional losses of light have been evaluated for the uniformly bright star (u=0), f^u, and the completely limb-darkened star (u=1), f^D, such that for arbitrary limb-darkening, the loss of light can be expressed as a weighted mean of the two extreme situations (9):

$$f = \frac{3(1-u)}{3-u} f^u + \frac{2u}{3-u} f^D \qquad (2.5)$$

Therefore it is possible to calculate the fractional losses of light from tables of f^u, f^D which have been extensively computed by Zessevitch (10) and Merrill (11). If we consider the individual luminosities of the two components normalized by $L_a + L_b = 1$, then the loss of light during any minimum can be computed from (9):

$$\begin{aligned} \ell_a &= (1-f_a(k,p))L_a + L_b \\ &= 1-f_a(k,p)L_a \\ \ell_b &= 1-f_b(k,p)L_b \qquad (2.6) \end{aligned}$$

depending upon whether the eclipse is an occultation or a transit.

2.3 THE ANALYSIS OF A LIGHT CURVE

The most difficult and basic problem confronting the analyst is to separate the light changes due to proximity effects from those due to the eclipses. However, in order to treat the method of analysis in an abbreviated form, consideration will be given only to systems of spherical stars sufficiently well separated as to introduce no auxilliary effects.

The fundamental problem of the eclipsing system can be reduced to two basic equations that under appropriate analysis will yield the unknown elements.

The first such equation arises from equating the value of δ as given by equation 2.1 with that given in equation 2.2. The result being

$$r_b{}^2(1+kp)^2 = \sin^2 \sin^2 i + \cos^2 i \qquad (2.7)$$

The second equation is found by comparing the normalized light loss (the fractional losses of light f^u, f^D normalized at the moment of internal tangency, $p = -1$) at different phases with the value of the depth of minima such that

$$\alpha(k,p) = \frac{f(k,p)}{f(k,-1)} = \frac{1-\ell}{1-\lambda}\,\alpha(k,p_0) = n\alpha_0$$

where $n = \dfrac{1-\ell}{1-\lambda}$

and λ is the luminosity at midminimum. The quantity $\alpha_0 = \alpha(k,p_0)$ is the maximum obscuration i.e. the maximum value of the normalized light loss at mid-eclipse while p_0 is the value of the geometrical depth of the minimum also at mid-eclipse.

The first attempt at a direct solution for the orbital elements was made by E.C. Pickering (12) on Algol (ß Persei). Despite Pickering's efforts, the problem remained a challenge until Henry Norris Russell proposed (13) an efficient and workable method in 1912.

(i) RUSSELL'S METHOD

The first fundamental equation 2.6 in the case of a total eclipse contains three unknowns k, r_b, and i which should be soluble by choosing three points on the light curve. Russell's idea was to eliminate r_b and i, then solve for k as the root of the determinental equation (9):

$$\begin{vmatrix} \sin^2{}_1 & (1+kp_1)^2 & 1 \\ \sin^2{}_2 & (1+kp_2)^2 & 1 \\ \sin^2{}_3 & (1+kp_3)^2 & 1 \end{vmatrix} = 0$$

Russell, in order to solve for k, assigned two of the points as specific values ($\alpha_1 = 0.6$, $\alpha_2 = 0.9$) while retaining the third as arbitrary. Expanding the determinant reveals that

$$\sin^2\theta = A + B\psi(k,p) \qquad (2.8)$$

$$A = \sin^2\theta_1$$

$$B = \sin^2\theta_1 - \sin^2\theta_2$$

$$\psi(k,\alpha) = \frac{2(p-p_1) + k(p^2-p_1{}^2)}{2(p_1-p_2) + k(p_1{}^2-p_2{}^2)}$$

The whole nature of the problem was then confined to the function $\psi(k,\alpha)$, but this was resolved by tabulating values for various degrees of limb darkening. Once the value of k had been chosen so equation 2.7 had been satisfied, the remaining elements were provided by:

$$\cot^2 i = \frac{B}{\phi_1(k)} - A$$

$$r_b^{\,2}\csc^2 i = \frac{B}{\phi_2(k)}$$

where $\phi_1(k)$, $\phi_2(k)$ were again tabulated functions which facilitated a more workable solution. The concept of tabulated functions originated with the Russell method and because of its usefulness was used by most subsequent investigators.

Despite the overall success achieved by Russell's method, certain defects existed centered around the use of the pivotal points. By selecting specific points, arbitrary weights were assigned to different parts of the light curve which made it impossible to ascertain the errors in the solution. To overcome this problem the concept of the iterative method was developed by Zdenek Kopal (14).

(ii) KOPAL'S METHOD

Again as a starting point, Kopal chose equation 2.6. At the moment of inner contact $p = -1$, the above equation reduces to

$$\sin^2\theta_{in}\sin^2 i + \cos^2 i = r_b^{\,2}(1-k^2) \qquad (2.9)$$

By solving for $\sin^2 \theta$ from equations 2.6 and 2.8, Kopal obtained the equation (14)

$$(p^2-1)C_1 + 2(p+1)C_2 + C_3 = \sin^2 \theta \quad (2.10)$$

$$C_1 = r_a^2 \csc^2 i$$

$$C_2 = r_a r_b \csc^2 i$$

$$C_3 = \sin^2 \theta in$$

Using a preliminary value of k and tables of $p(k,\alpha)$ equation 2.9 could be solved by least squares using each point on the light curve. A comparison of the initial value of k to $k = C_1/C_2$ determined whether or not another iteration need be performed. If not one could solve for the unknowns from

$$r_a = \sqrt{C_1} \sin i$$

$$r_b = \frac{C_2 \sin i}{\sqrt{C_1}}$$

$$\cot^2 i = \frac{(C_2 - C_1)^2}{C_1} - C_3$$

The use of Kopal's technique as a by-product of the least squares analysis provided the weighting of each element and the probable errors incurred.

2.4 METHOD OF KITAMURA

The preceding procedures for the analysis of light curves proved disadvantageous in two areas. These were:

(1) it was impossible to know prior to the evaluation of the elements which of the two minima was due to an occultation and which was due to a transit.

(2) the method of analysis was different for the different types of eclipses (partial, total, annular) which in themselves could be difficult to distinguish by mere inspection of the light curve.

These difficulties were overcome by the recent development of a new method as suggested by Masatoshi Kitamura (15,16). This method of analysis is dependent upon certain characteristic functions of the eclipse derived from integral transforms of the ascending and descending branches of each minimum.

As in previous methods, the distribution of surface brightness is assumed to follow the cosine law given by eq. 2.4. The luminosity of the system during an eclipse is given by

$$\ell(\theta) = 1-L_1 f(\theta) \qquad (2.11)$$

where as before the fractional loss of light is given by equation 2.5. Two types of integral transforms, the incomplete Fourier transforms are defined as (15)

$$S_n = \int_o^{\phi} \ell(\theta) \sin n\theta d\theta$$

$$C_n = \int_o^{\phi} \ell(\theta) \cos n\theta d\theta$$

ϕ is the phase angle of an observed point outside of eclipse and n is zero or an integer. Another pair of transforms can be formulated as (15)

$$s_n = \int_o^{\theta_o} f(\theta) \sin n\theta d\theta$$

$$c_n = \int_o^{\theta_o} f(\theta) \cos n\theta d\theta$$

θ_o is the phase angle at the moment of first contact given by

$$\tan^2\theta_o = \frac{\sin^2 i}{1 - (r_a^2 + r_b^2)} - 1$$

By substitution of equation 2.11, a relation can be found between S_n and s_n, and C_n and c_n:

$$L_1 s_n = \frac{1-\cos n\phi}{n} - S_n \qquad n = 1,2,\ldots.$$

$$= 0 \qquad n = o$$

$$L_1 c_n = \frac{\sin n\phi}{n} - C_n \qquad n = 1,2,\ldots$$

$$= \phi - C_o \qquad n = o$$

The conversion of the observed light curve into its incomplete Fourier transforms can be performed numerically with the aid of an electronic computer. The problem of finding a solution reduces to the simultaneous determination of the geometrical elements of the eclipse r_a, r_b and i. In order to solve for these three unknowns, three independent relations are required (15):

(1) ratio between $L_1 s_n$, $L_1 c_n$ for specified values of n

$$F_n = \frac{L_1 s_n}{L_1 c_n} = \frac{s_n}{c_n}$$

(2) the ratio between $L_1 c_o$ and the depth of minimum, λ,

$$E = \frac{L_1 c_o}{1-\lambda}$$

(3) combination of both minima

$D_p = (L_1 c_o)p$ p = primary

$D_s = (L_1 c_o)s$ s = secondary

solving for luminosities

$$L_p = \frac{D_p}{(c_o)p} \quad , \quad L_s = \frac{D_s}{(c_o)s}$$

using the normalizing condition $L_p + L_s = 1$ then

$$\frac{D_p}{(c_o)_p} + \frac{D_s}{(c_o)_s} = 1$$ D_p, D_s are deduced from the observations

while $(c_o)_p$, $(c_o)_s$ are given by theory.

The three functions F_n, E, c_o are now considered as the characteristic functions of the eclipse. The solution for r_a, r_b, and i in principle follows from a comparison of the values of the functions E and F_n deduced from the observations for either minimum with their theoretical values as functions of the geometrical elements and use of the third relation to establish the best set of elements. The same procedure is applied to any type of eclipse.

The type of eclipse occurring at each minimum is obtained by consideration of the theoretical nature of the eclipse and the mathematical properties of the functions discussed

in section 2-2. Kitamura has shown that

$$\left[F_n\right]_{tr} \leq \left[F_n\right]_{oc}$$

$$\left[E\right]_{tr} \leq \left[E\right]_{oc}$$

where the equal sign holds for

(1) grazing eclipse

(2) $u_a = u_b = 0$

(3) $r_a = r_b$, $u_a = u_b$

Because of the advantages enjoyed by this method and its application to computer analysis, it was chosen as the procedure to be used in the examination of EI CEPHEI.

CHAPTER 3

OBSERVATIONAL EQUIPMENT AND EXPERIMENTAL TECHNIQUE

3.1 OBSERVING EQUIPMENT

The observations of EI CEPHEI were carried out at the Devon Observatory located approximately twelve miles from the city of Edmonton, Alberta. The equipment housed there and used throughout the study of EI CEPHEI includes:

12½" reflecting telescope and single channel photometer.
high voltage d.c. voltage power supply
digital voltmeter and digital clock
automatic recorder - printer
multi-band radio

The acquisition of the four-band radio in early 1973 permitted monitoring of the digital display clock by comparison with transmitted radio timing signals. The physical construction of the photometer was altered slightly during the course of observing. It was noted that different readings were being obtained when the telescope was shifted to the opposite side of its mounting pier. This effect was attributed to a sagging of the photometer in the collar used to couple it to the telescope. To rectify this situation, three supporting members constructed from metal rods were attached to the photometer and telescope to ensure a rigid structure.

(i) TELESCOPE

The telescope used throughout the observing sequence was a Cassegrainian - type reflector equipped with an equatorial mount. This instrument was obtained in 1967 from Tinsley Laboratories, Berkeley, California.

The primary mirror is 12.5 inches in diameter with a focal length of 48 inches while the secondary mirror is 3.75 inches in diameter. The initial installation of the equipment brought with it certain mechanical problems. A close examination of these problems and methods used to correct them are described by B. Brown (17).

(ii) PHOTOMETER

The basic components constituting the photometer are:
field eyepiece and assembly
diaphragm
diaphragm - viewing telescope and assembly
filter assembly
Fabry lens
photomultiplier

An outline of the basic construction and a description of the individual components and their use is again given by B. Brown (17). Although the photometer is essentially the same as that

described in the above reference, certain changes have been made.

(a) photomultiplier

In previous observations with the photometer an RCA or Sylvannia 1P21 photomultiplier tube was used. This cell was characterized by an entrance window on the side which meant the photomultiplier had to be mounted perpendicular to the main photometer housing. In its place was substituted an E.M.I. 9502S end-on photomultiplier cell which can be placed directly into the photometer housing.

(iii) FILTERS

Previous multicolour photometric work at Devon was carried out in the UBV filter system developed by Johnson and Morgan(10).

Observations on EI CEPHEI were done, however, using the intermediate band filters of the "uvby" system. This system was developed by B. Stromgren(18) who describes it as an extension to the MK (Morgan-Keenan) classifications and UBV photometry. The use of intermediate band filters allows a two dimensional classification of stars founded upon particular spectral indices which are based on the measures of selected spectral features.

Intermediate band photometry is characterized by filters which have a half-width of 100 Å - 300 Å. In particular, the filters defining the system in use are:

(a) u - glass filter (8mm UG11 + 1mm WG3)
  - centered about 3500Å
  - transmission region: - above atmospheric cut off but below Balmer discontinuity

(b) v - interference filter
  - centered about 4100Å
  - above region of crowding of hydrogen lines as they approach the Balmer discontinuity
  - contains the line Hδ

(c) b - interference filter
  - centered about 4700Å
  - contains no strong spectral features

(d) y - interference filter
  - centered about 5500Å
  - contains no strong spectral features

Three separate indices have been developed for use with this system. They are the colour index (b-y) and the two colour differences $m_1 = (v-b) - (b-y)$ and $c_1 = (u-v) - (v-b)$.

The colour index (b-y) displays properties similar to the B-V of the UBV system but is influenced less by metallic absorption lines. The colour difference m_1 is a good indicator of metal content. The other colour difference c_1 acts as a good index to the Balmer discontinuity.

It has been found that the "uvby" system displays no effects on band-width such as extinction, transformation or interstellar reddening and is also uninfluenced by strong spectral features other than $H\delta$, Balmer discontinuity and the line blanketing. The system is filter defined and hence depends little on the atmosphere of the Earth or the photocell in use.

(iv) Miscellaneous

The remaining equipment was obtained from Hewlett Packard and consists of:

3340A Digital Voltmeter

J74562A Digital Recorder

Harrison 6525A DC Power Supply
(0-4000V, 0-5mA)

The operational use and limitations of each instrument are

described in detail in the appropriate operating manuals provided by the manufacturer.

3.2 SYSTEMATICS

The first accurate catalogue of stars compiled by Hipparchus around 135 B.C. and later refined by Ptolemy to form the ALMAGEST listed all objects in five magnitude intervals. The brightest stars were designated as m = 1 with those stars just visible to the naked eye as m = 6. An investigation by Sir John Herschel pointed out that the difference between a first magnitude star and a sixth magnitude star corresponded to an intensity ratio of approximately one hundred. N. Pogson in 1856 proposed a system whereby the intensity ratio was taken to be exactly one hundred such that each magnitude interval corresponded to $(100)^{.2}$ or 2.512 in intensity. Pogson's definition together with Fechner's psycho-physical law of sensation (sensation $\propto$log stimulus) led to the modern system of photometry known as the Pogson Scale which states

$$m = -2.5 \log \text{luminosity} \qquad 3.1a$$

$$\text{or } m_2 - m_1 = 2.5 \log (\ell_1/\ell_2). \qquad 3.1b$$

A measure of the observed intensities of two stars should therefore give an accurate evaluation of their magnitude difference. However in reality direct measures obtained in any

photometric study are functions of

(1) parameters related to the equipment

(2) the atmosphere through which the light must travel

(3) the wavelength of the light

The observer must therefore treat his data in such a manner as to make allowances for all effects not directly attributable to the object under study. The reduction of the data consists of two major steps:

(1) an adjustment for atmospheric extinction

(2) a transformation to a standard system of magnitudes and colours.

(i) EXTINCTION

During its passage through the Earth's atmosphere, the starlight is reduced in intensity by absorption or through scattering processes. The degree of intensity loss will be directly proportional to the thickness of the atmosphere to be traversed. The relation that governs the rate at which the light is diminshed in magnitude is

$$m_{obs} = m_{star} - kX \qquad 3.2$$

Here X, the path length, is expressed in units of air mass at the zenith of the observer and k, the extinction coefficient is a measure of the light loss in magnitudes for a star also at the zenith. From equation 3.2, a value for the extinction coefficient can be obtained from the slope of a graph of observed magnitude versus the air mass. To a first approximation, a value for the relative air mass is given by the secant of the zenith distance, z. From geometrical considerations the value of sec z is

$$\sec z = (\sin\delta\sin\phi + \cos\delta\cos\phi\cos h)^{-1}$$

in which ϕ is the observer's latitude while δ and h are the declination and hour angle of the star. If a more exact value is required$^{(2)}$, X may be obtained from the polynomial expansion

$$\begin{aligned} X = \sec z - {} & 0.018167\ (\sec z - 1) - \\ & 0.002875\ (\sec z - 1)^2 - \\ & 0.0008083\ (\sec z - 1)^3 \end{aligned}$$

As mentioned before the "uvby" system incorporates three indices b-y, m_1, and c_1. Each of these values is individually affected by the presence of extinction and each must accordingly be corrected. The resulting extinction equations

have a form identical to 3.2

$$y = y_{obs} - kx$$
$$(b-y) = (b-y)_{obs} - k_1x$$
$$m_1 = m_{1\,obs} - k_2x$$
$$c_1 = c_{1\,obs} - k_3x$$

From investigations carried out by Crawford and Barnes$^{(19)}$ at the Kitt Peak National Observatory, it has been found that mean values of the extinction coefficients are adequate for all but the most uncertain of nights and that no second order extinction effects are present.

(ii) TRANSFORMATION

The light received by the photometer will be characterized by a certain wavelength and band-width. The response of the photometer to this radiation will be dependent upon its optical components and the spectral sensitivity of the cathode. Therefore each combination of telescope, filters, and photomultiplier will define its own natural colour system. In order for a set of observations to be of use to another observer with a different natural system, it is necessary to transform all measurements to a universally accepted standard system.

For ideal black-bodies, the colour indices measured on different natural systems are transformable by a simple linear equation,

$$C' = \mu C + Ƶ_c$$

whereas for magnitudes the equation has the form,

$$m' = m + \varepsilon C + Ƶ_m$$

where ε, μ are "scale" constants and $Ƶc$, $Ƶm$ are "zero-point" constants. For the filter system in use for this study, the transformation equations are applied to the three indices b-y, m_1, c_1 and one magnitude, y which corresponds closest to the V of the UBV system. The equations are

$$V = A + B\ (b-y) + y_{obs}$$

$$(b-y) = C + D\ (b-y)_{obs}$$

$$m_1 = E + Fm_1(obs) + J(b-y)$$

$$c_1 = G + Hc_1(obs) + I(b-y)$$

In their definition of the standard system for "uvby" photometry, Crawford and Barnes provide a list of carefully studied stars with accurately measured values of V, (b-y), m_1, and c_1. Therefore from an accurate investigation of a group of

standard stars, a series of equations can be formulated and the unknown coefficients obtained by means of a least squares technique. As in the instance of the extinction coefficients, mean values for the transformation coefficients are found to be sufficient for most observing nights.

3.2b DIFFERENTIAL PHOTOMETRY

The application of both types of corrections is important when stars are studied as individual sources by means of absolute photometry. However, in the case of variable stars certain advantages are realized by applying the techniques of differential photometry. Differential photometry relies on a comparison of a variable with an immediately adjacent comparison star known not to be variable. If the comparison star is close enough in position and colour, then a difference in magnitudes as demonstrated by equation 3.1b will eliminate all first order extinction effects. In addition to the variable and its comparison star, periodic observations of a check star are also included to ensure that there are no undetected fluctuations in the comparison star. The extinction equations become

$(b_v - y_v) = (b_v - y_v)_{obs} - k_1 x$ for the variable and

$(b_c - y_c) = (b_c - y_c)_{obs} - k_1 x$ for the comparison.

By taking a difference we obtain

$$(b'-y') = (b'-y')_{obs} - k_1 \Delta x$$

and if the comparison star has been chosen judiciously so that Δx can be assumed negligible then $(b'-y') = (b'-y')_{obs}$

where $$b' = b_v - b_c$$

$$y' = y_v - y_c$$

Simplification of the transformation equations can also be achieved by the same procedure. If V_v is the transformed value of the visual magnitude for the variable and V_c that of the comparison star then

$$V_v - V_c = V' = A + B(b_v - y_v) + y_v - A - B(b_c - y_c) - y_c$$

$$= y'_{obs} + B(b'-y')$$

In summary, the use of differential photometry reduces both extinction and transformation equations to the following:

(1) $V' = y'_{obs}$

(2) $(b'-y') = (b'-y')_{obs}$

(3) $m_1' = m_1'(obs)$

(4) $c_1' = c_1'(obs)$

(5) $V = y\ obs + B\ (b'-y')_{obs}$

(6) $(b - y) = (b' - y')_{obs}$

(7) $m_1 = Fm_1'(obs) + J(b' - y')_{obs}$

(8) $c_1 = Hc_1'(obs) + I(b' - y')_{obs}$

A complete and detailed paper on the principles of the above mentioned corrections together with a variety of methods of solution is presented by R. Hardie.(20)

3.2c TIME

In the discussion of the eclipsing binary it was noted that the two components of the light-curve were the observed magnitude versus a time scale. Of these two quantities, the most accurately measured is time. For the study of a variable, the time for each measurement is converted to Universal Time, the time at which an observer in Greenwich would observe the same event. This is accomplished by adding or subtracting the number of hours ahead of or behind Greenwich time as designated by the Standard Time Zone. Consideration must be given to the implementation of daylight saving time where necessary. The hours, minutes, and seconds are then converted to a decimal fraction of a day. Each day is denoted by a running number, specified by the Julian Day, counted from January 1, 4713 B.C. The Julian Day begins at noon Greenwich

time so that the night hours fall on the same Julian Day.

(i) LIGHT TIME

As the Earth moves around the sun in its orbit, the distance from the Earth to the variable is constantly changing. For a star on the ecliptic, the starlight reaches the Earth between eight minutes earlier or eight minutes later than it reaches the sun due to its finite velocity. This cyclic variation in arrival times has little effect on variables with periods of a few days but is readily detectable in short period stars. To eliminate this phenomenon a small correction is added to the Julian date to convert it to Heliocentric time, the time that the light would reach the sun. The mathematical relation(2) governing the heliocentric correction is

$$\Delta t = -0^{d}.005770 \quad (AX + BY)$$

$$X = R \cos \odot$$

$$Y = R \sin \odot \cos \varepsilon$$

$$A = \cos \delta \cos \alpha$$

$$B = \tan \varepsilon \sin \delta + \cos \delta \sin \alpha$$

The above relations involve the following quantities

δ, α, declination and right ascension of the star

ε angle of the ecliptic 23°27'

R earth - sun distance normalized to one astronomical unit

$\odot$ position of earth on the ecliptic referred to the vernal equinox

The multiplicative constant $0^{d}.005770$ is the time, given as a decimal of a day, taken by a beam of light to travel a distance of one astronomical unit. The numerical values for R, o for each day of the year together with the value of the corresponding Julian Day are listed in THE AMERICAN EPHEMERIS AND NAUTICAL ALMANAC.

3.3a OBSERVATIONAL TECHNIQUES:

Prior to any observing, the time as registered by the digital clock was set to the standard mean solar time. This was first accomplished by the use of a common watch which had been checked prior to leaving for the observatory. However, with the arrival of the radio, the clock was matched to timing signals broadcast by radio station WWV operated by the United States National Bureau of Standards. For work on EI CEP and the comparison star A, the photometer was operated at 1900 volts d.c. Each filter reading consisted of a series of individual voltmeter measurements approximately 12 or more in number. A complete set of readings for one star involved eight separate filter measurements in the sequence uvbyybvu. After each filter had been positioned, a check was made to see if the star under observation was still located in the center of the isolating diaphragm; if not steps were taken to recenter

it. Therefore an entire series took from three to five minutes to complete depending upon the amount of readjusting of the telescope. After each set of data, filter readings were taken of the surrounding sky. Included in the observing schedule were occasional measurements of the check star B.

3.3b REDUCTION OF OBSERVATIONAL DATA

Each set of filter readings was checked to eliminate any missed printings or measurements that appeared to have been printed incorrectly. The automatic printer recorded two voltage readings per second and if two consecutive readings for the same time appeared, one reading was eliminated. After all necessary corrections had been made, averages were taken for each filter reading. Then another average between two identical filters were made. From this value was subtracted the sky reading for the appropriate filter to give the observed intensities. These values for EI CEP and A were then used in equation 3.1b to calculate the observed magnitude change for the variable. Since a complete observation of one star took a finite length of time, the time of observation was taken as the midpoint of the set. Because of the length of the orbital period, the preliminary reduction used to plot the light curve

CHAPTER 4

THE OBSERVATIONS

4.1 The Observations of EI CEP and Comparison Star.

The observing program for EI CEP which began in August, 1972, and which was terminated in October, 1973, resulted in 208 individual observations for each of the four filters. These represent measurements taken on twenty-five separate nights. Although EI CEP was observed on many occasions, due to the length of the eclipses and the changing nature of the sky conditions, no one minimum was followed completely during the course of a single night. A list of the differential magnitudes for each filter in the sense COMPARISON STAR - VARIABLE together with the differential values of the colour index (b-y), and the colour differences m_1 and c_1 are presented in Table 4.1. The observations are listed in order of the corresponding phase which was computed from the values of the period and the epoch of primary conjunction cited by Abrami (TABLE 1.1). All the times of observations were corrected to heliocentric time. The estimated values of the errors in the measurements of phase and for the magnitudes are listed below the corresponding title. Graphs of Δm versus phase were plotted to yield the light curve for each filter.

TABLE 4.1

OBSERVATIONS OBTAINED ON EI CEPHEI

PHASE	u	v	b	y	b-y	m_1	c_1
±.0003	±.030	±.025	±.025	±.025			
.0009	-1.053	-.846	-1.067	-1.220	.153	.068	-.428
.0020	-1.062	-.870	-1.049	-1.207	.158	.021	-.370
.0034	-1.083	-.818	-1.053	-1.224	.171	.064	-.500
.0042	-0.945	-.788	-1.017	-1.116	.099	.129	-.385
.0053	-1.013	-.849	-1.033	-1.163	.129	.055	-.348
.0078	-0.903	-.748	-0.901	-1.045	.144	.008	-.308
.0096	-0.958	-.771	-0.911	-1.087	.175	-.035	-.326
.0114	-0.920	-.715	-0.865	-0.999	.134	.016	-.354
.0117	-0.826	-.614	-0.782	-0.981	.199	-.031	-.380
.0128	-0.910	-.679	-0.831	-0.979	.148	.004	-.383
.0138	-0.830	-.579	-0.800	-0.976	.176	.044	-.472
.0142	-0.842	-.606	-0.862	-0.924	.062	.192	-.492
.0155	-0.739	-.616	-0.772	-0.888	.116	.040	-.279
.0169	-0.695	-.548	-0.721	-0.859	.138	.035	-.319
.0183	-0.678	-.531	-0.723	-0.830	.107	.084	-.338
.0194	-0.770	-.533	-0.669	-0.839	.170	-.033	-.373
.0198	-0.669	-.548	-0.664	-0.823	.159	-.044	-.237
.0217	-0.845	-.463	-0.650	-0.776	.126	.061	-.570
.0220	-0.632	-.443	-0.639	-0.805	.166	.030	-.384
.0240	-0.623	-.443	-0.610	-0.846	.236	-.070	-.346
.0240	-0.621	-.466	-0.654	-0.763	.108	.081	-.345
.0254	-0.606	-.482	-0.626	-0.870	.244	-.100	-.268
.0263	-0.653	-.427	-0.595	-0.720	.124	.045	-.395
.0269	-0.673	-.465	-0.691	-0.814	.123	.103	-.433
.0284	-0.598	-.398	-0.624	-0.782	.158	.068	-.426
.0295	-0.536	-.347	-0.576	-0.721	.146	.083	-.419
.0297	-0.545	-.391	-0.565	-0.583	.018	.156	-.327

.0304	-0.639	-.364	-0.634	-0.733	.099	.170	-.544
.0319	-0.567	-.383	-0.515	-0.727	.212	-.080	-.317
.0333	-0.614	-.344	-0.490	-0.721	.231	-.084	-.418
.0353	-0.655	-.411	-0.571	-0.759	.188	-.027	-.405
.0359	-0.577	-.423	-0.583	-0.674	.091	.069	-.315
.0376	-0.647	-.423	-0.573	-0.734	.162	-.012	-.374
.0396	-0.605	-.414	-0.520	-0.746	.227	-.120	-.298
.0459	-0.518	-.346	-0.531	-0.728	.197	-.013	-.357
.0477	-0.467	-.429	-0.650	-0.681	.031	.198	-.258
.0495	-0.583	-.429	-0.595	-0.765	.170	-.004	-.320
.0514	-0.572	-.413	-0.585	-0.732	.147	.025	-.332
.0532	-0.529	-.484	-0.595	-0.790	.195	-.084	-.156
.0562	-0.559	-.363	-0.581	-0.837	.256	-.038	-.413
.0724	-0.603	-.422	-0.593	-0.760	.166	.005	-.353
.0748	-0.615	-.417	-0.587	-0.718	.131	.039	-.368
.0785	-0.569	-.396	-0.573	-0.788	.215	-.038	-.350
.0808	-0.587	-.395	-0,579	-0.721	.142	.043	-.377
.0838	-0.651	-.461	-0.614	-0.768	.155	-.002	-.342
.1904	-0.587	-.402	-0.566	-0.669	.103	.061	-.349
.1917	-0.632	-.389	-0.606	-0.715	.109	.108	-.460
.1934	-0.541	-.393	-0.552	-0.728	.176	-.017	-.306
.1954	-0.497	-.309	-0.647	-0.746	.099	.239	-.526
.2233	-0.578	-.405	-0.574	-0.752	.177	-.008	-.342
.2258	-0.577	-.401	-0.541	-0.731	.190	-.050	-.316
.2283	-0.562	-.412	-0.570	-0.771	.201	-.043	-.307
.2309	-0.566	-.386	-0.563	-0.747	.184	-.007	-.357
.2328	-0.569	-.392	-0.561	-0.723	.163	.005	-.345
.3290	-0.553	-.363	-0.585	-0.686	.101	.121	-.411
.3307	-0.553	-.380	-0.552	-0.702	.150	.022	-.345
.3331	-0.553	-.377	-0.572	-0.747	.174	.021	-.371
.3354	-0.582	-.394	-0.595	-0.758	.163	.038	-.389
.3423	-0.577	-.356	-0.529	-0.706	.177	-.004	-.394
.3451	-0.577	-.353	-0.525	-0.686	.161	.011	-.396
.4468	-0.517	-.289	-0.531	-0.798	.268	-.026	-.470
.4495	-0.602	-.381	-0.591	-0.786	.195	.014	-.431

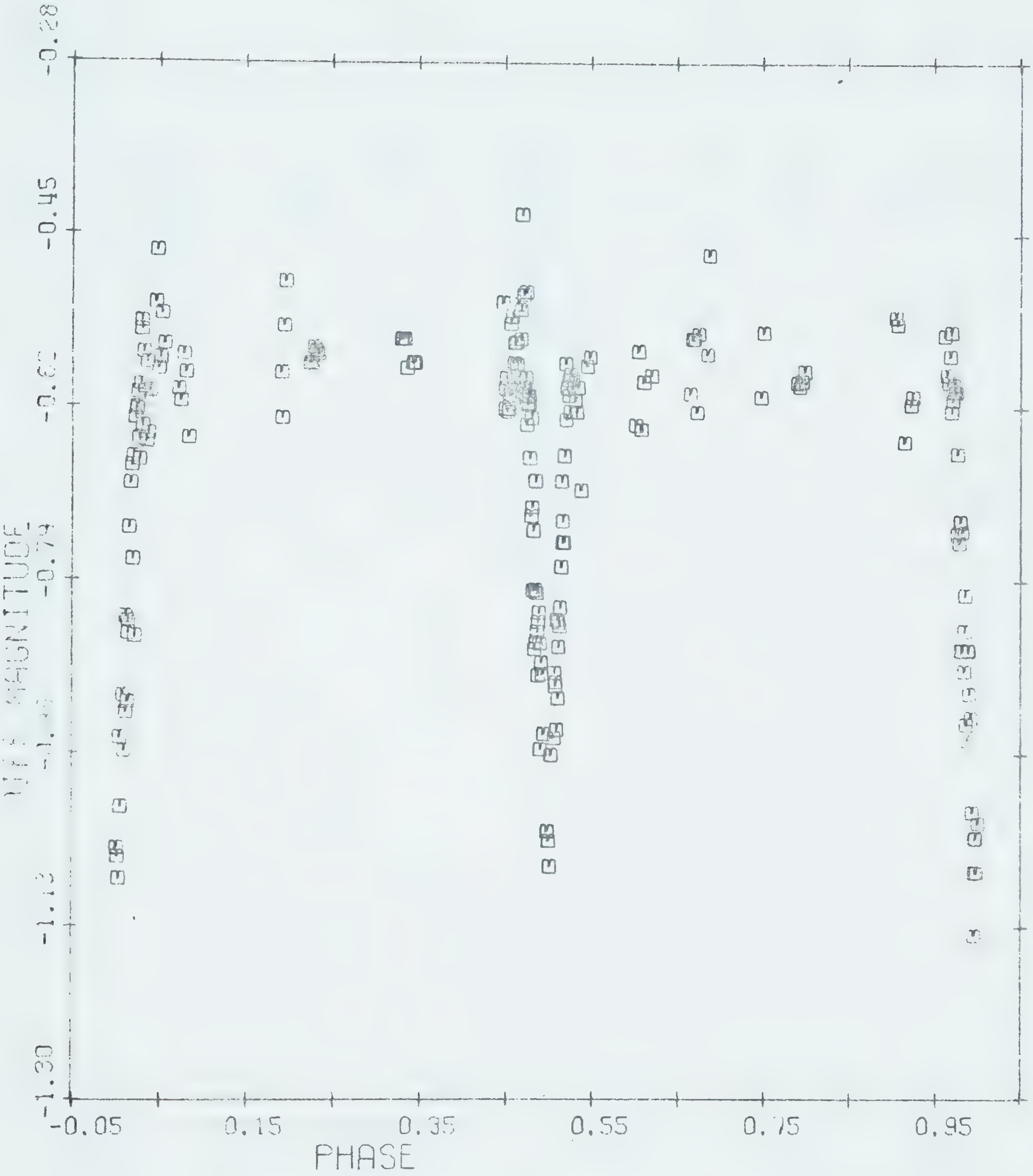
.4495	-0.622	-.422	-0.610	-0.760	.150	.038	-.387
.4503	-0.592	-.407	-0.583	-0.720	.137	.040	-.360
.4519	-0.625	-.375	-0.580	-0.751	.171	.034	-.454
.4536	-0.622	-.422	-0.588	-0.722	.134	.032	-.365
.4540	-0.608	-.364	-0.545	-0.716	.171	.010	-.425
.4553	-0.538	-.397	-0.563	-0.711	.148	.019	-.307
.4566	-0.608	-.405	-0.561	-0.725	.164	-.008	-.358
.4571	-0.531	-.390	-0.551	-0.724	.172	-.011	-.301
.4593	-0.577	-.401	-0.558	-0.705	.148	.009	-.332
.4594	-0.597	-.396	-0.582	-0.769	.187	-.002	-.387
.4602	-0.557	-.340	-0.522	-0.736	.215	-.033	-.399
.4624	-0.556	-.392	-0.528	-0.675	.147	-.012	-.300
.4625	-0.581	-.373	-0.550	-0.702	.152	.025	-.385
.4626	-0.578	-.410	-0.540	-0.737	.197	-.067	-.298
.4644	-0.519	-.338	-0.532	-0.688	.156	.038	-.375
.4651	-0.589	-.381	-0.550	-0.728	.178	-.010	-.376
.4666	-0.616	-.429	-0.575	-0.755	.179	-.033	-.335
.4670	-0.553	-.373	-0.561	-0.691	.129	.060	-.369
.4672	-0.524	-.346	-0.570	-0.725	.154	.070	-.401
.4685	-0.605	-.394	-0.577	-0.707	.130	.054	-.395
.4686	-0.430	-.378	-0.558	-0.728	.170	.011	-.233
.4694	-0.603	-.372	-0.560	-0.705	.145	.044	-.420
.4703	-0.509	-.388	-0.545	-0.636	.091	.066	-.279
.4706	-0.600	-.367	-0.527	-0.707	.180	-.020	-.394
.4718	-0.600	-.364	-0.586	-0.692	.106	.116	-.459
.4725	-0.607	-.380	-0.557	-0.685	.128	.050	-.404
.4729	-0.590	-.392	-0.583	-0.734	.151	.040	-.389
.4736	-0.507	-.411	-0.576	-0.664	.088	.077	-.260
.4739	-0.637	-.408	-0.568	-0.708	.139	.021	-.390
.4748	-0.616	-.447	-0.645	-0.776	.131	.067	-.366
.4755	-0.609	-.480	-0.556	-0.784	.228	-.152	-.205
.4756	-0.608	-.410	-0.574	-0.652	.078	.086	-.361
.4768	-0.617	-.442	-0.616	-0.747	.131	.044	-.350
.4771	-0.609	-.531	-0.637	-0.797	.159	-.054	-.183
.4774	-0.670	-.455	-0.638	-0.805	.166	.017	-.398

.4793	-0.726	-.479	-0.665	-0.817	.152	.035	-.433
.4794	-0.631	-.411	-0.715	-0.780	.065	.240	-.525
.4807	-0.718	-.590	-0.681	-0.881	.200	-.109	-.218
.4809	-0.800	-.505	-0.737	-0.820	.083	.149	-.527
.4822	-0.741	-.461	-0.659	-0.936	.276	-.078	-.478
.4828	-0.857	-.580	-0.774	-0.879	.105	.088	-.470
.4841	-0.799	-.616	-0.833	-1.046	.213	.004	-.399
.4846	-0.692	-.525	-0.723	-0.930	.207	-.009	-.365
.4848	-0.850	-.629	-0.776	-0.925	.149	-.003	-.368
.4861	-0.802	-.569	-0.753	-0.942	.189	-.004	-.418
.4864	-0.840	-.621	-0.792	-0.911	.119	.052	-.390
.4874	-0.830	-.731	-0.842	-0.997	.155	-.044	-.210
.4876	-0.883	-.681	-0.824	-1.036	.211	-.069	-.344
.4883	-0.821	-.674	-0.878	-0.965	.087	.117	-.352
.4899	-0.956	-.753	-0.876	-1.042	.165	-.043	-.325
.4893	-0.852	-.674	-0.874	-0.997	.123	.078	-.378
.4912	-0.870	-.680	-0.798	-0.994	.196	-.077	-.309
.4921	-0.882	-.820	-0.921	-1.073	.153	-.052	-.162
.4942	-0.941	-.764	-1.020	-1.077	.057	.199	-.432
.4986	-1.037	-.792	-0.963	-1.131	.168	.003	-.416
.5001	-1.046	-.840	-1.014	-1.138	.125	.049	-.380
.5011	-1.071	-.855	-1.044	-1.145	.101	.089	-.406
.5026	-0.961	-.808	-0.988	-1.080	.092	.089	-.334
.5061	-0.945	-.758	-0.928	-1.046	.118	.051	-.356
.5066	-0.880	-.682	-0.847	-1.040	.193	-.028	-.363
.5077	-0.894	-.716	-0.874	-1.027	.153	.004	-.335
.5082	-0.890	-.653	-0.829	-0.989	.160	.017	-.413
.5090	-0.937	-.741	-0.847	-1.006	.159	-.053	-.301
.5096	-0.829	-.683	-0.867	-1.018	.151	.033	-.329
.5106	-0.905	-.667	-0.907	-1.059	.152	.089	-.479
.5110	-0.831	-.693	-0.842	-0.985	.143	.006	-.287
.5110	-0.855	-.656	-0.843	-0.962	.119	.068	-.387
.5127	-0.835	-.607	-0.861	-0.944	.084	.170	-.481
.5130	-0.816	-.644	-0.785	-0.915	.129	.012	-.313
.5145	-0.777	-.576	-0.736	-0.920	.184	-.024	-.360

.5150	-0.692	-.619	-0.751	-0.901	.151	-.019	-.206
.5161	-0.753	-.518	-0.726	-0.913	.187	.021	-.443
.5162	-0.732	-.545	-0.703	-0.860	.157	.001	-.345
.5178	-0.753	-.451	-0.697	-0.935	.248	-.011	-.538
.5181	-0.668	-.509	-0.653	-0.823	.170	-.026	-.303
.5196	-0.577	-.525	-0.692	-0.865	.173	-.005	-.220
.5202	-0.632	-.474	-0.612	-0.783	.171	-.032	-.297
.5210	-0.601	-.482	-0.619	-0.781	.162	-.026	-.256
.5230	-0.609	-.475	-0.586	-0.757	.171	-.061	-.244
.5231	-0.593	-.427	-0.631	-0.783	.152	.052	-.371
.5245	-0.624	-.375	-0.582	-0.741	.159	.048	-.457
.5249	-0.587	-.463	-0.570	-0.744	.173	-.066	-.231
.5281	-0.593	-.423	-0.556	-0.733	.177	-.044	-.303
.5309	-0.614	-.432	-0.596	-0.725	.129	.035	-.346
.5325	-0.625	-.379	-0.572	-0.699	.128	.066	-.439
.5341	-0.600	-.381	-0.581	-0.746	.166	.034	-.419
.5373	-0.702	-.387	-0.568	-0.723	.155	.026	-.496
.5444	-0.579	-.387	-0.518	-0.749	.231	-.099	-.323
.5482	-0.570	-.391	-0.591	-0.732	.141	.059	-.379
.6017	-0.637	-.409	-0.534	-0.745	.211	-.086	-.353
.6047	-0.565	-.387	-0.543	-0.715	.172	-.016	-.333
.6075	-0.641	-.427	-0.584	-0.708	.124	.034	-.372
.6107	-0.595	-.430	-0.579	-0.740	.161	-.012	-.313
.6195	-0.588	-.469	-0.582	-0.711	.129	-.016	-.232
.6652	-0.607	-.409	-0.584	-0.721	.137	.038	-.373
.6674	-0.553	-.391	-0.599	-0.755	.156	.051	-.368
.6700	-0.553	-.377	-0.530	-0.688	.158	-.005	-.328
.6726	-0.625	-.383	-0.559	-0.717	.158	.018	-.417
.6745	-0.547	-.403	-0.563	-0.730	.167	-.007	-.304
.6848	-0.568	-.451	-0.574	-0.714	.140	-.016	-.241
.6870	-0.470	-.350	-0.513	-0.644	.131	.031	-.282
.7476	-0.610	-.388	-0.611	-0.759	.148	.075	-.445
.7503	-0.546	-.389	-0.531	-0.718	.187	-.045	-.298
.7906	-0.595	-.421	-0.566	-0.733	.167	-.022	-.320
.7930	-0.599	-.376	-0.558	-0.696	.137	.045	-.406

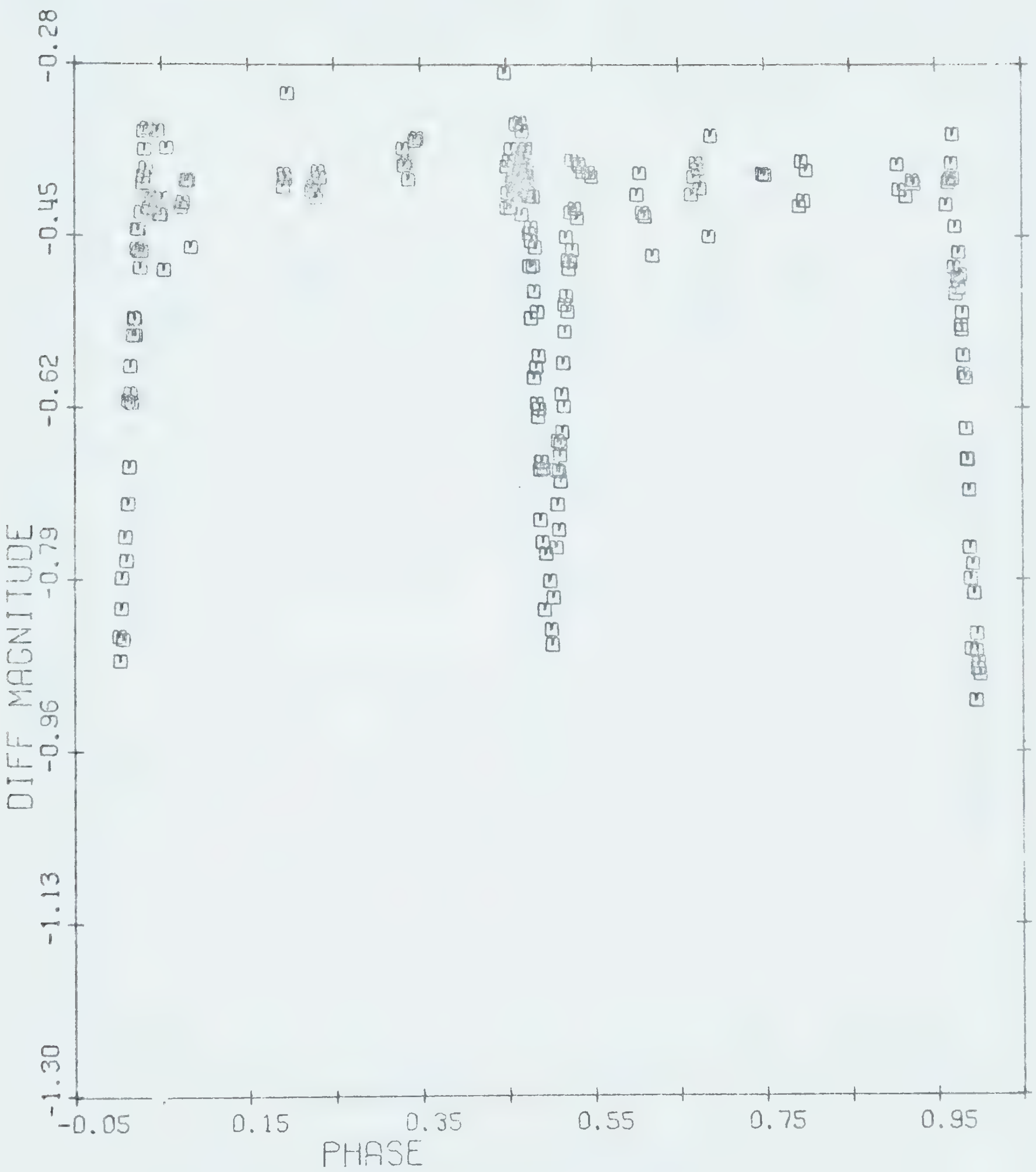
.7957	-0.594	-.415	-0.561	-0.676	.115	.031	-.325
.7983	-0.584	-.385	-0.548	-0.714	.166	-.003	-.362
.9049	-0.531	-.379	-0.634	-0.754	.120	.135	-.406
.9071	-0.538	-.405	-0.593	-0.685	.092	.097	-.322
.9150	-0.653	-.411	-0.566	-0.810	.244	-.089	-.397
.9226	-0.617	-.395	-0.564	-0.733	.168	.001	-.391
.9244	-0.609	-.398	-0.575	-0.753	.178	-.002	-.386
.9618	-0.548	-.419	-0.546	-0.718	.172	-.046	-.255
.9645	-0.587	-.397	-0.557	-0.682	.125	.035	-.350
.9666	-0.595	-.394	-0.546	-0.660	.115	.037	-.353
.9679	-0.569	-.378	-0.548	-0.728	.180	-.010	-.361
.9695	-0.545	-.350	-0.505	-0.725	.220	-.064	-.351
.9696	-0.624	-.393	-0.508	-0.779	.270	-.155	-.346
.9712	-0.612	-.482	-0.616	-0.744	.128	.006	-.265
.9721	-0.597	-.441	-0.634	-0.775	.141	.053	-.349
.9728	-0.604	-.508	-0.622	-0.765	.142	-.028	-.210
.9748	-0.604	-.491	-0.630	-0.716	.086	.052	-.251
.9765	-0.665	-.466	-0.641	-0.805	.164	.011	-.374
.9765	-0.743	-.502	-0.696	-0.878	.182	.012	-.435
.9787	-0.752	-.489	-0.735	-0.841	.105	.141	-.511
.9794	-0.732	-.539	-0.722	-0.887	.165	.018	-.375
.9798	-0.731	-.543	-0.729	-0.893	.164	.022	-.373
.9807	-0.858	-.526	-0.753	-0.891	.139	.087	-.558
.9814	-0.742	-.569	-0.752	-0.937	.185	-.002	-.355
.9822	-0.858	-.587	-0.763	-0.923	.160	.016	-.446
.9844	-0.878	-.591	-0.828	-0.993	.164	.073	-.524
.9845	-0.839	-.642	-0.814	-0.982	.168	.004	-.369
.9854	-0.804	-.672	-0.801	-0.974	.173	-.044	-.260
.9864	-0.931	-.672	-0.852	-1.008	.156	.024	-.439
.9879	-0.946	-.703	-0.938	-1.117	.179	.056	-.479
.9884	-0.858	-.759	-0.925	-1.013	.089	.077	-.264
.9894	-0.858	-.790	-0.923	-1.019	.096	.037	-.201
.9901	-0.900	-.860	-1.011	-1.164	.153	-.002	-.192
.9921	-0.924	-.775	-0.975	-1.192	.217	-.017	-.349
.9937	-1.017	-.805	-0.995	-1.194	.199	-.009	-.403

.9957	-1.139	-.911	-1.080	-1.229	.149	.020	-.397
.9958	-1.078	-.861	-1.027	-1.215	.188	-.023	-.380
.9967	-1.043	-.844	-1.063	-1.184	.121	.098	-.417
.9977	-1.043	-.879	-1.076	-1.232	.156	.041	-.360
.9987	-1.077	-.872	-1.037	-1.248	.212	-.048	-.369
.9999	-1.028	-.885	-1.060	-1.216	.156	.019	-.318



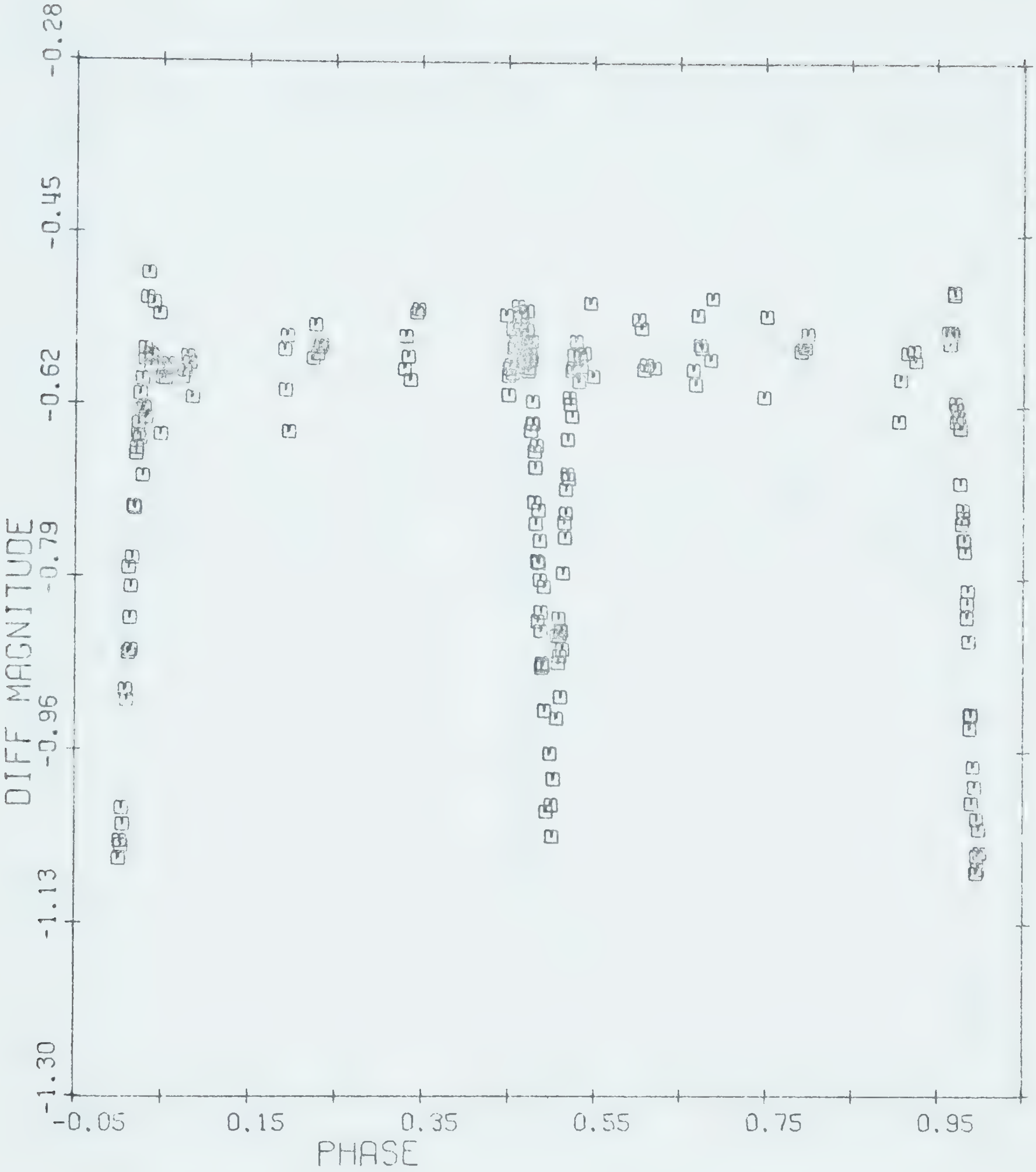
LIGHT CURVE OF EI CEPHEI U-FILTER

FIGURE 4.1



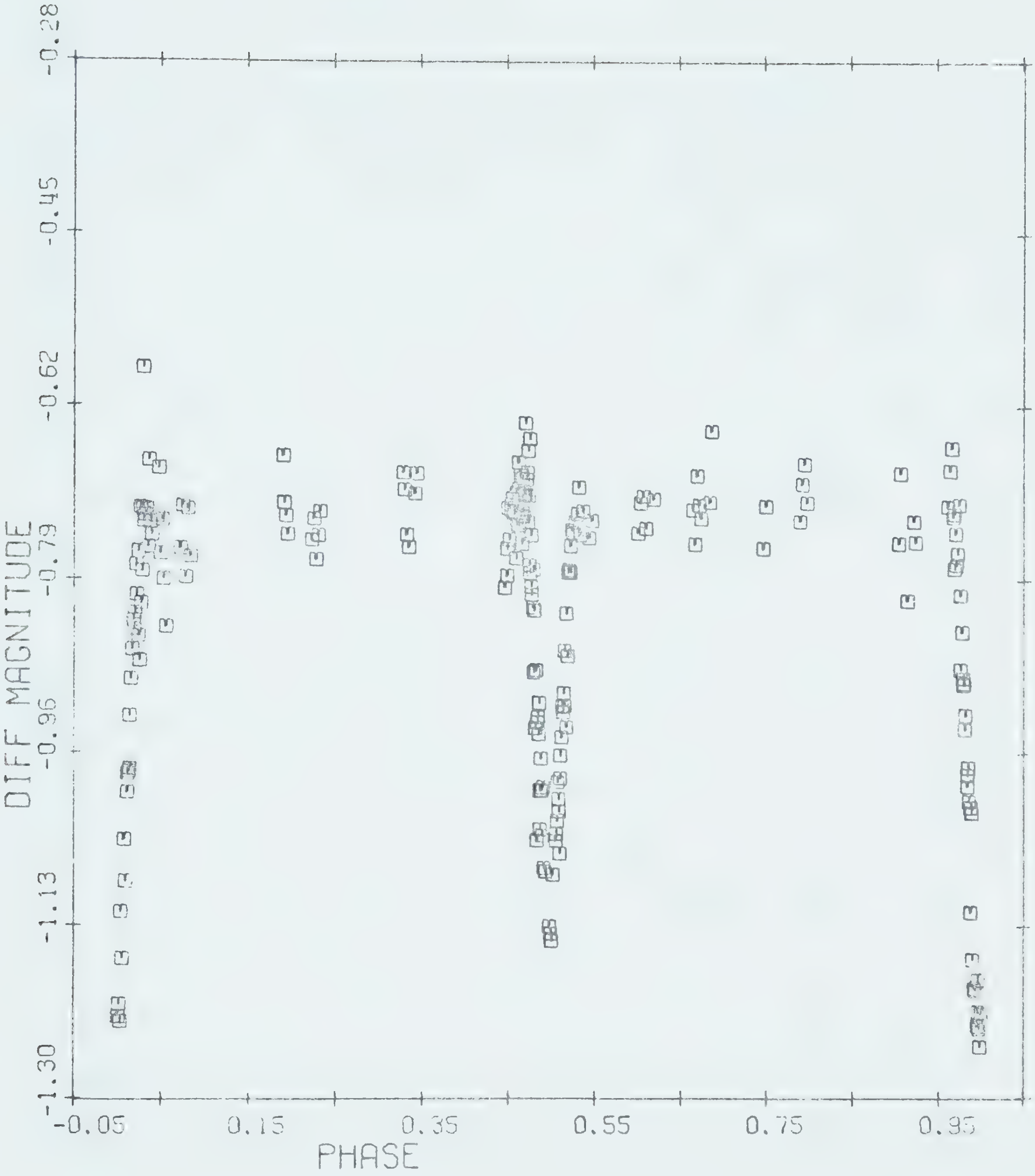
LIGHT CURVE OF EI CEPHEI V-FILTER

FIGURE 4.2



LIGHT CURVE OF EI CEPHEI B-FILTER

FIGURE 4.3

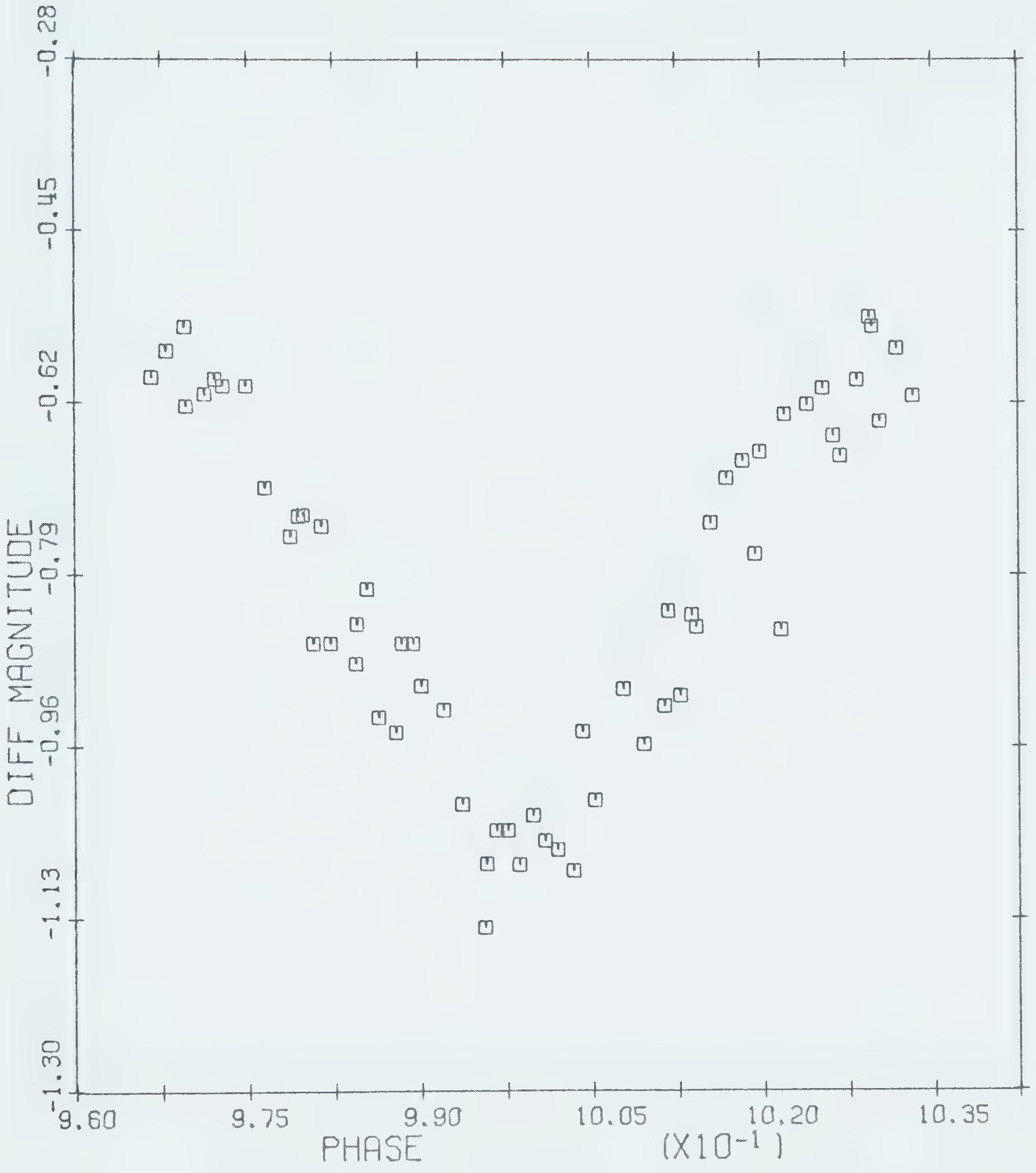


LIGHT CURVE OF EI CEPHEI Y-FILTER

FIGURE 4.4

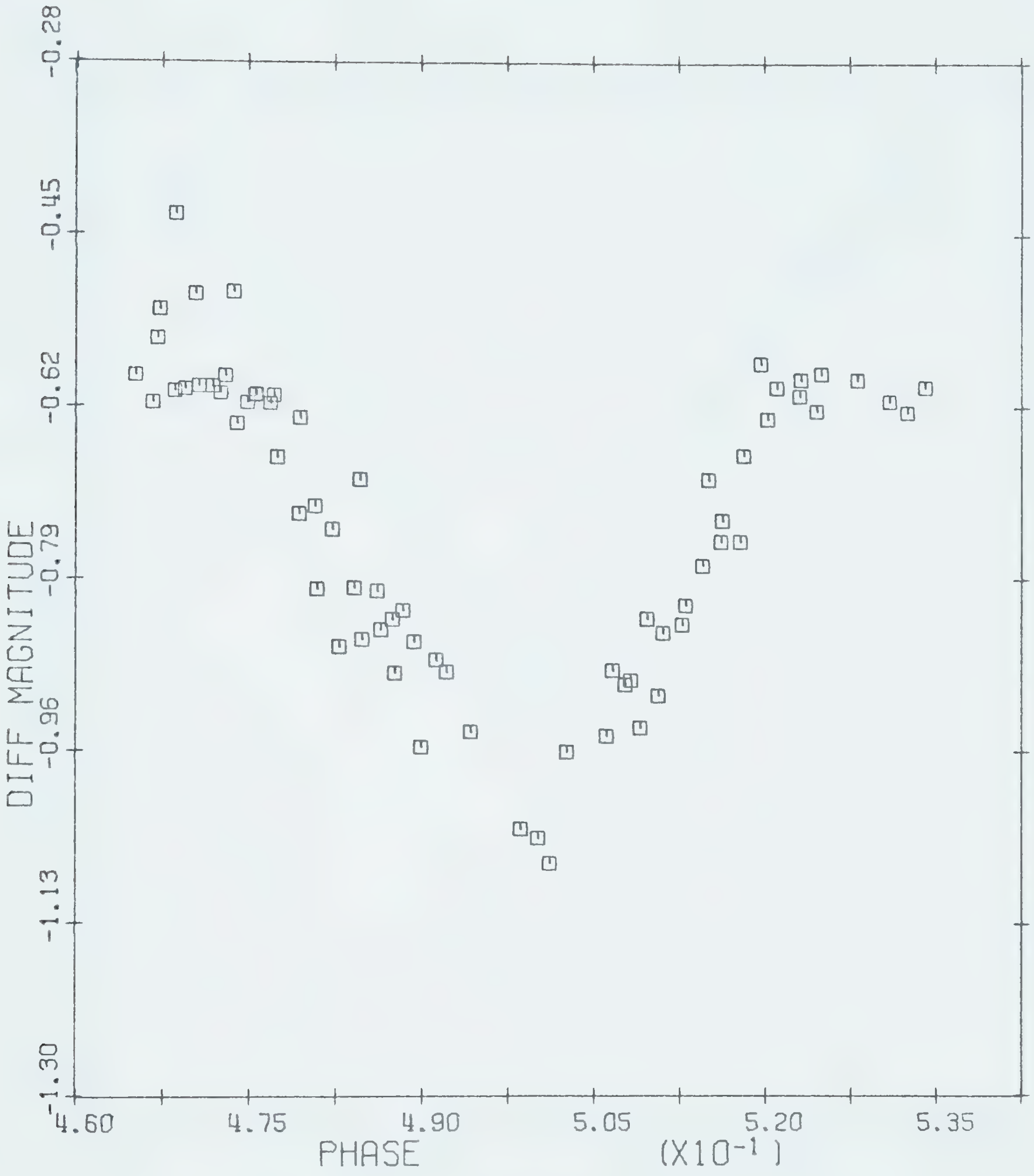
(i) The Light Curves

Upon inspection, the light curves display two well defined minima of slightly different depths that occur one half an orbital period apart thus indicating little or no change in the values of P and t_o calculated by Abrami. Superimposed upon the minima and readily noticeable outside eclipse is the presence of a high degree of scatter in the measurements. This scatter is present in varying degrees in all four filters and probably arises from poor sky conditions and poor observational techniques. A re-examination of the data and the elimination of measurements of two nights of observing failed to reduce the scatter appreciably. A night-by-night plot of the secondary minima indicated that the scatter was to be found randomly distributed among all observing nights. No effects due to reflection or ellipticity of the individual constituents, noticeable outside of eclipse, could be detected since the scatter effectively disguised any non-eclipse variations. Another feature of the light curves is the apparent lower value of Δm at the shoulder of the ascending branch of the secondary minimum. In order to examine this feature more closely, the individual minima have been isolated and expanded and are presented in the next



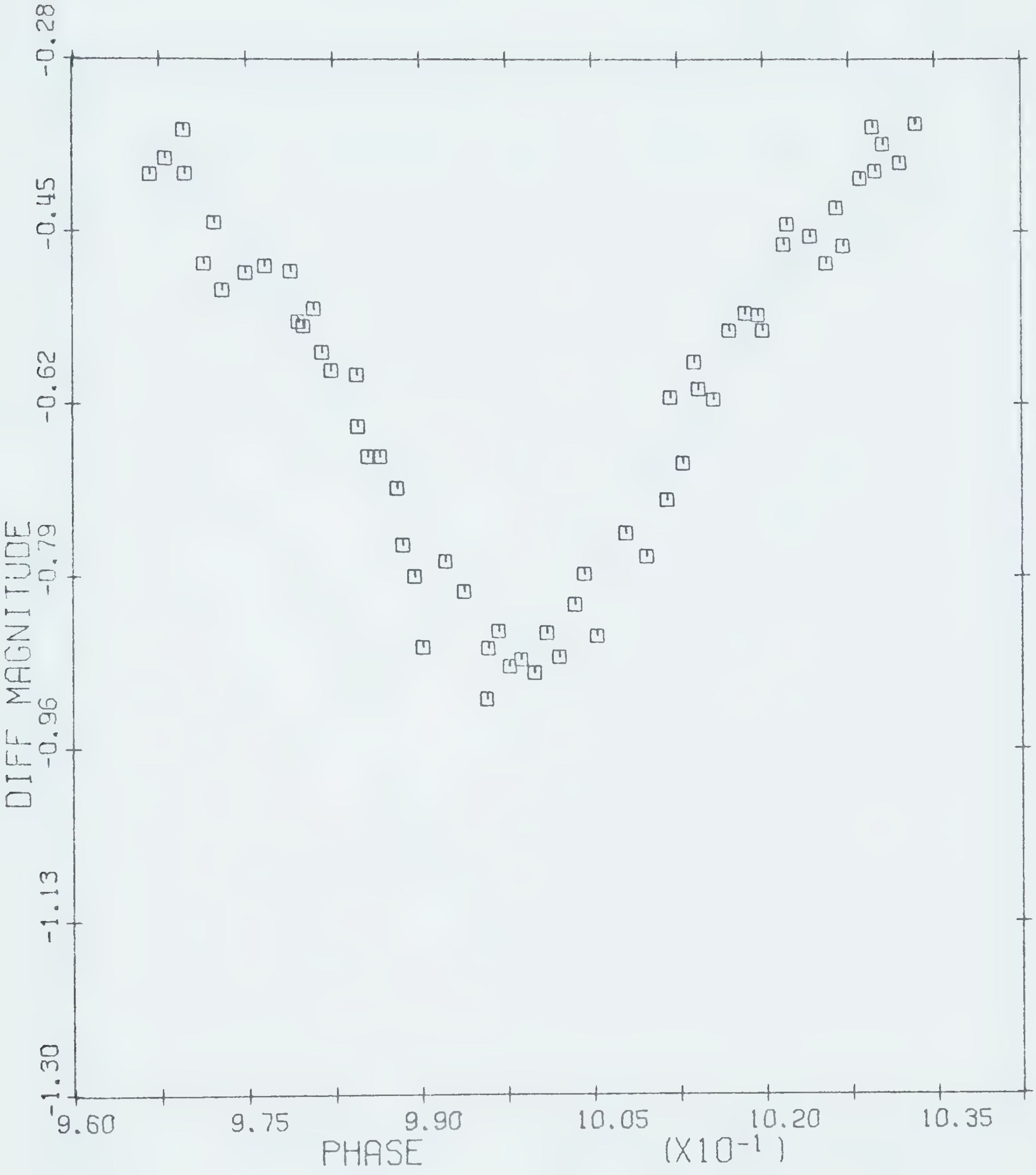
PRIMARY MINIMUM U-FILTER

FIGURE 4.5



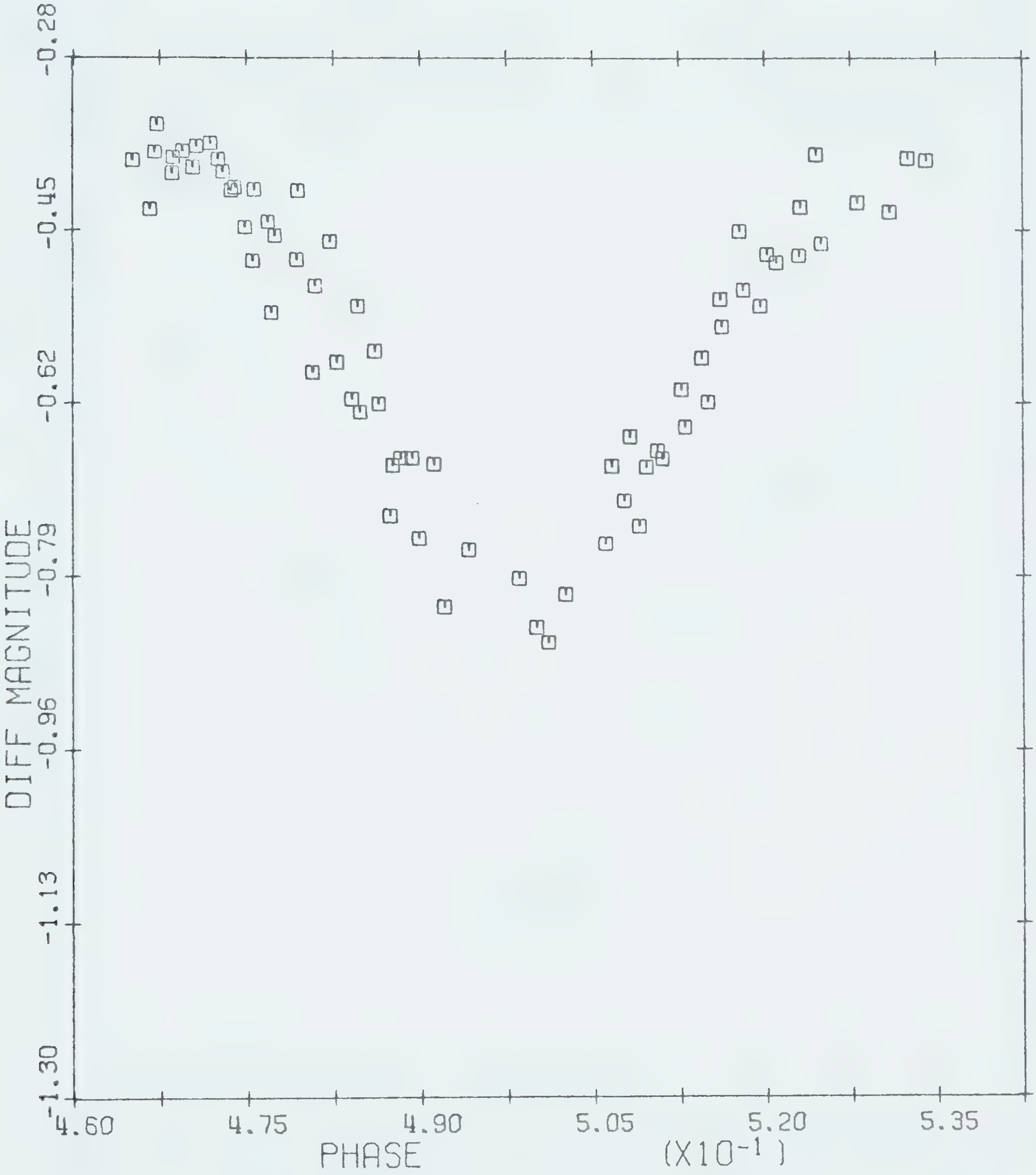
SECONDARY MINIMUM U-FILTER

FIGURE 4.6



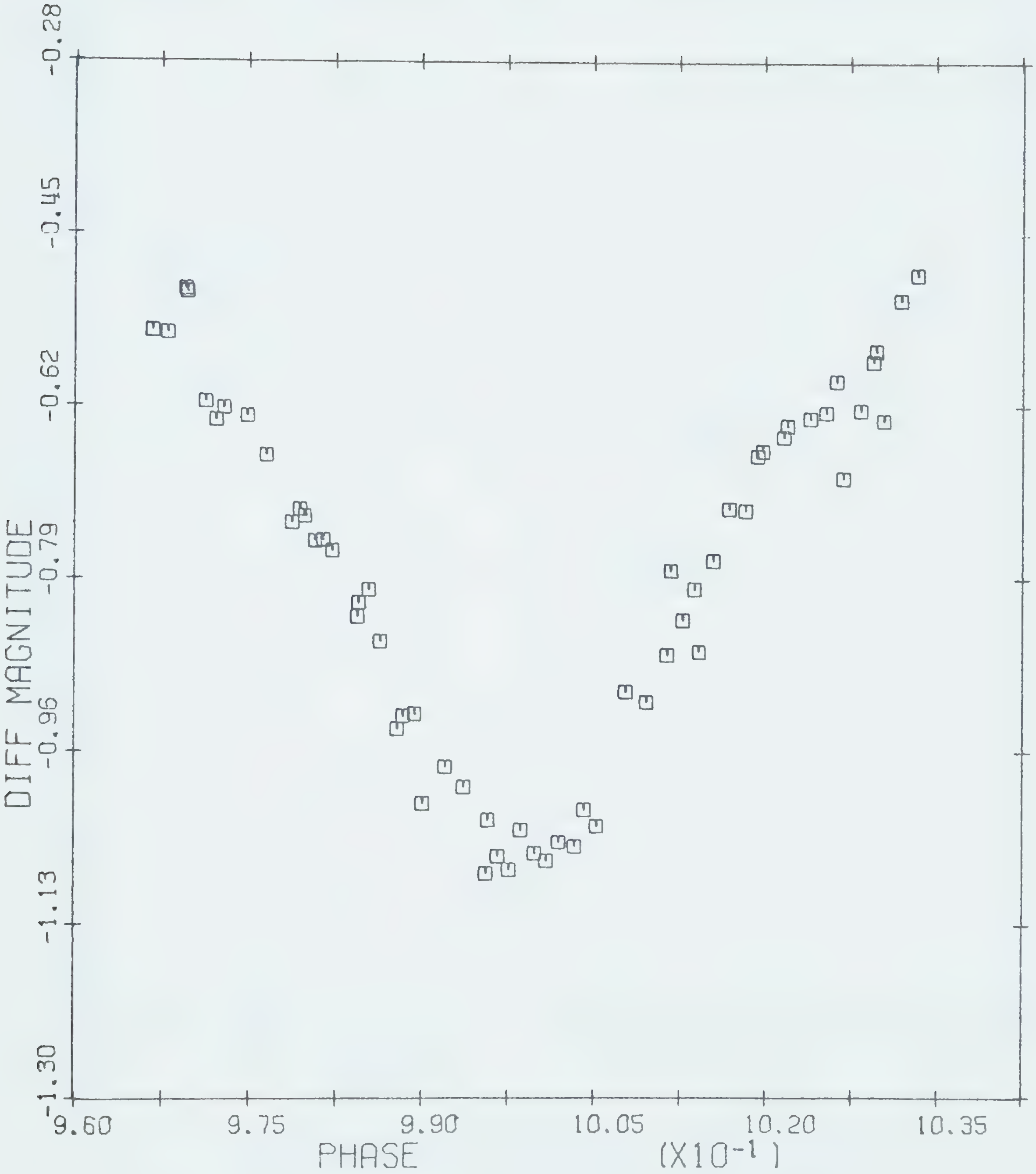
PRIMARY MINIMUM V-FILTER

FIGURE 4.7



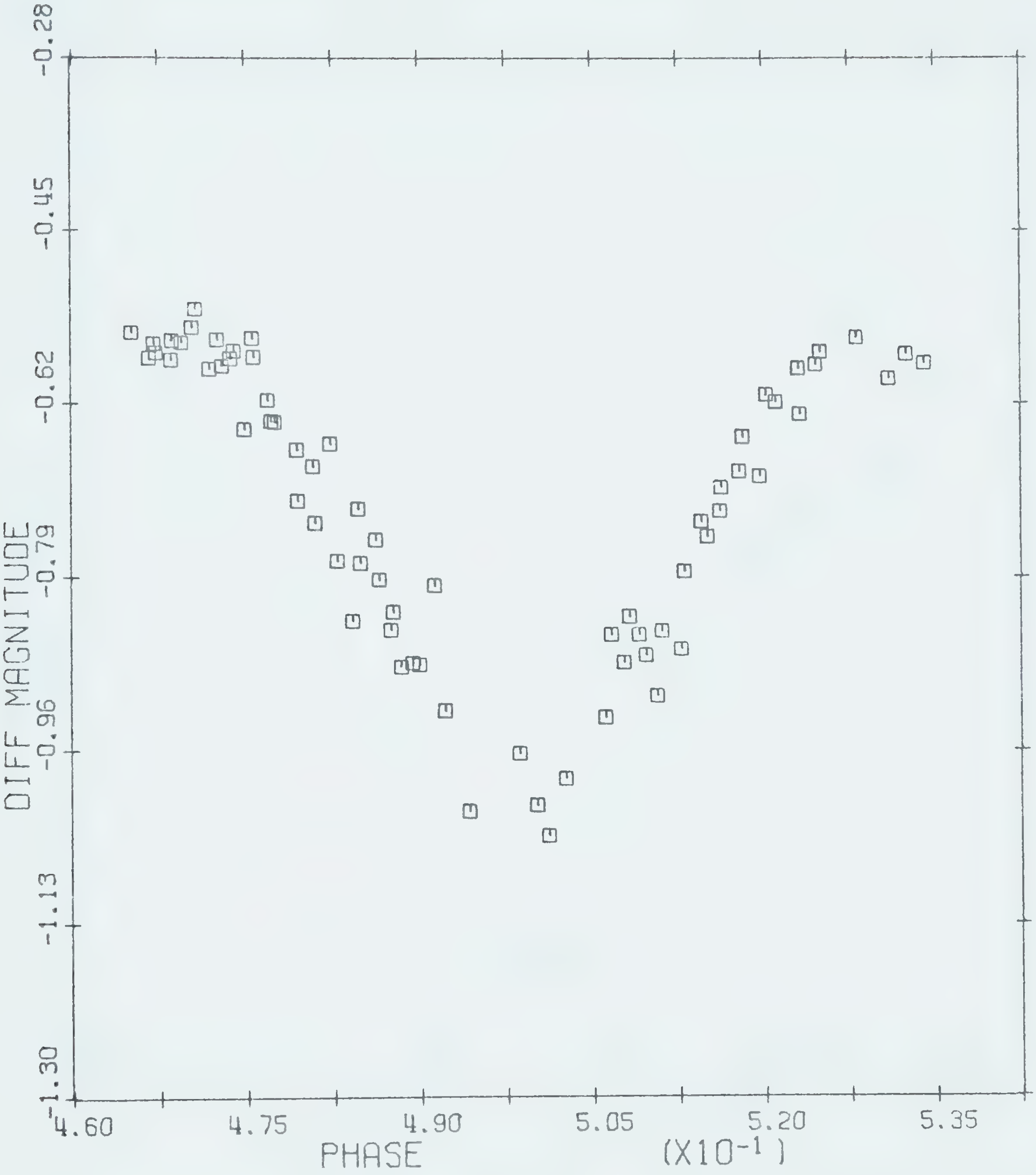
SECONDARY MINIMUM V-FILTER

FIGURE 4.8



PRIMARY MINIMUM B-FILTER

FIGURE 4.9



SECONDARY MINIMUM B-FILTER

FIGURE 4.10

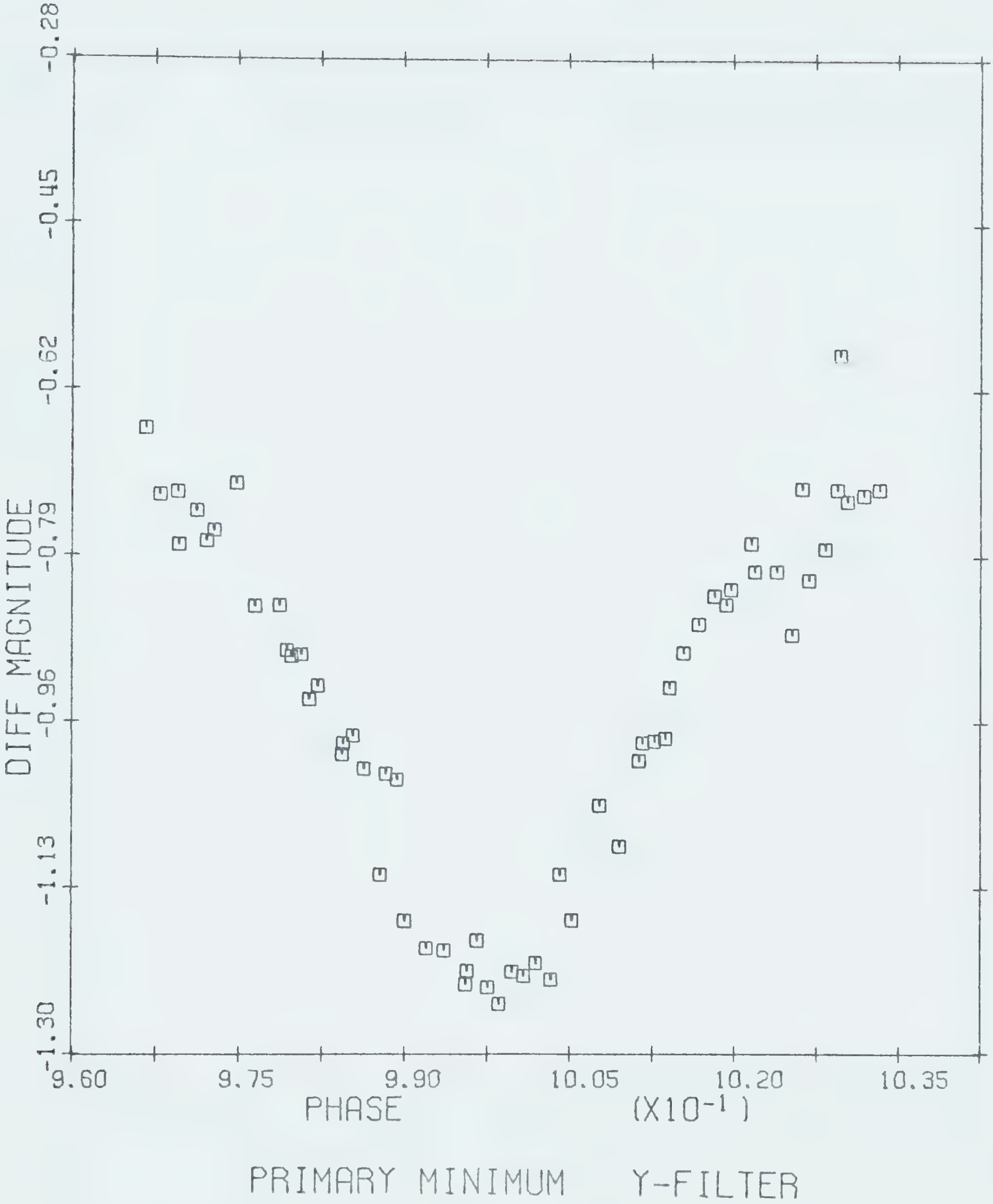
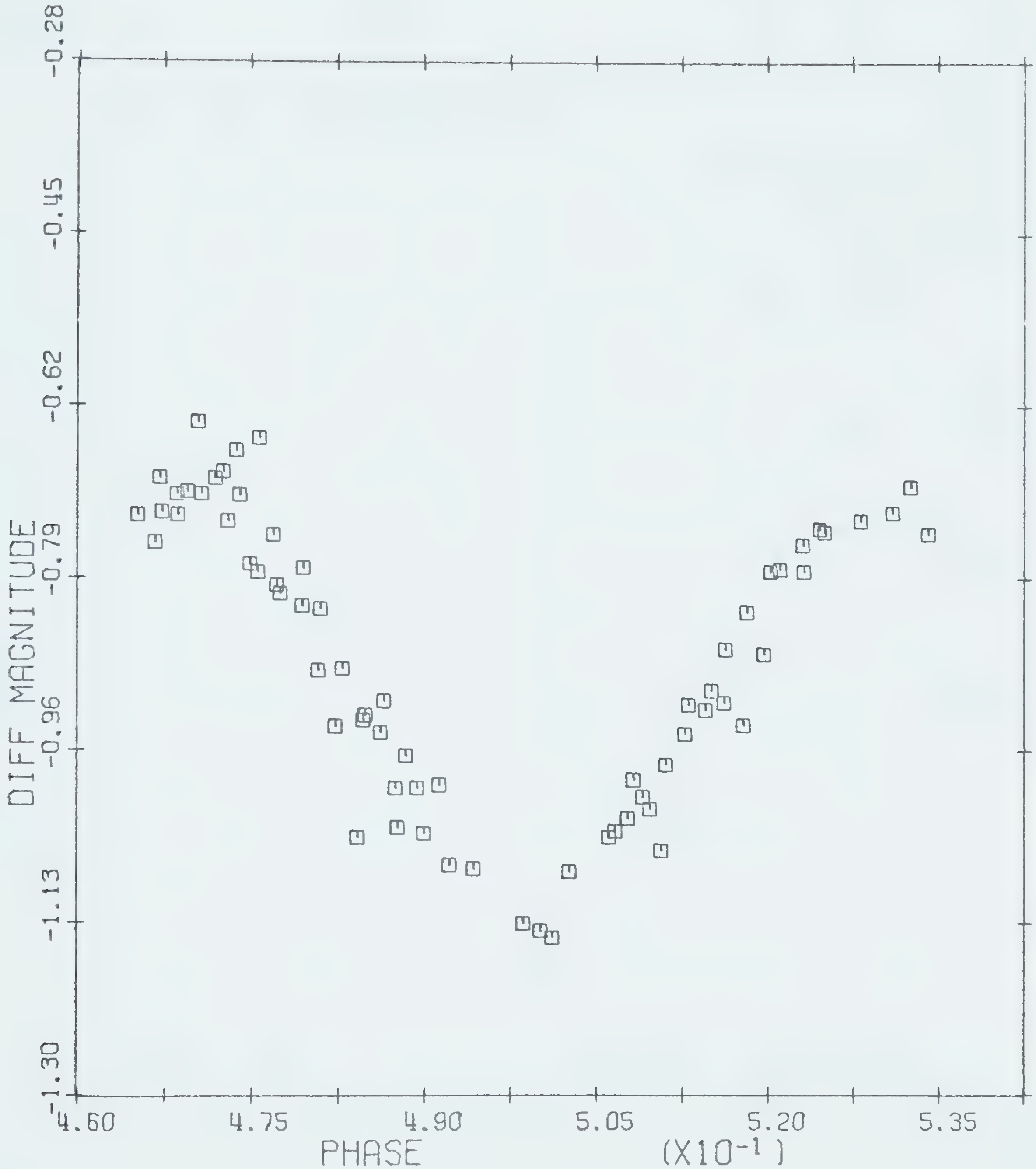

-0.28
-0.45
-0.62
-0.79
-0.96
-1.13
-1.30
DIFF MAGNITUDE
9.60
9.75
9.90
10.05
10.20
10.35
PHASE
(X10^{-1})
PRIMARY MINIMUM Y-FILTER

FIGURE 4.11



SECONDARY MINIMUM Y-FILTER

FIGURE 4.12

graphs

(ii) The Individual Minima

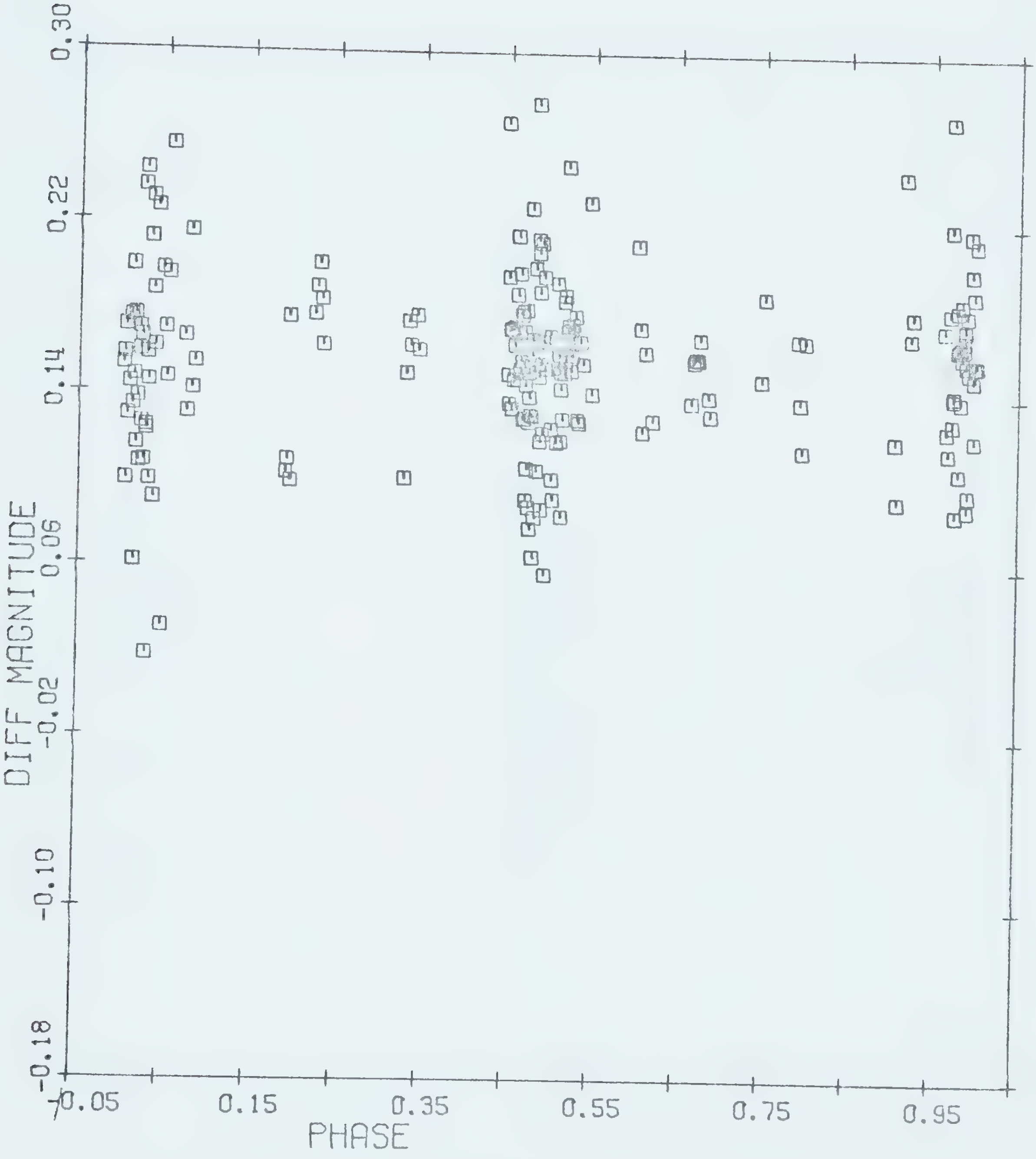
The secondary minima are to some extent non-symmetrical especially in the y filter. A lack of symmetry is also prevelant in the primary minima to a lesser degree except for the b filter. The ascending branches of the secondary minima appear not to reach the level outside eclipse enjoyed prior to the descending branch. It should be noted however, that more observations were made around the descending branch than the ascending branch. Because of the presence of the previously mentioned scatter and a lack of additional observations, it is difficult to determine whether or not these peculiarities in the minima are physical phenomenon caused by presence of gas between the components or through some unknown mechanism or are merely a result of the inaccuracy in the data points themselves. In his photo-electric study, Abrami notes that his light curve shows no extra eclipse variation. However this work was carried out using an unfiltered photometer and as such is less sensitive to any spectral phenomenon that might be associated with circumstellar gas. Popper also makes no mention in his spectroscopic analysis of any spectral features which might be indicative of the presence of any surrounding gas.

(iii) The Colour Indices

The next three figures are plots of (b-y), m_1, and c_1 against phase. The graph of (b-y) shows the least amount of scatter while those of m_1 and c_1 exhibit progressively more. Part of this increase can be attributed to the addition of an extra factor in both m_1 and c_1 which increases the largest expected error from .050 mag. for (b-y) to .100 mag. for m_1 and .110 mag for c_1. Each graph still displays a larger degree of scatter than might be anticipated especially during minima. Values of the mean and standard deviation were calculated and listed in TABLE 4.2 for different sections of the light curve as a test of the amount of scatter. For the colour difference c_1, the values of the standard deviation are somewhat smaller between eclipse, whereas for the other two indices the distinction is not as easily seen.

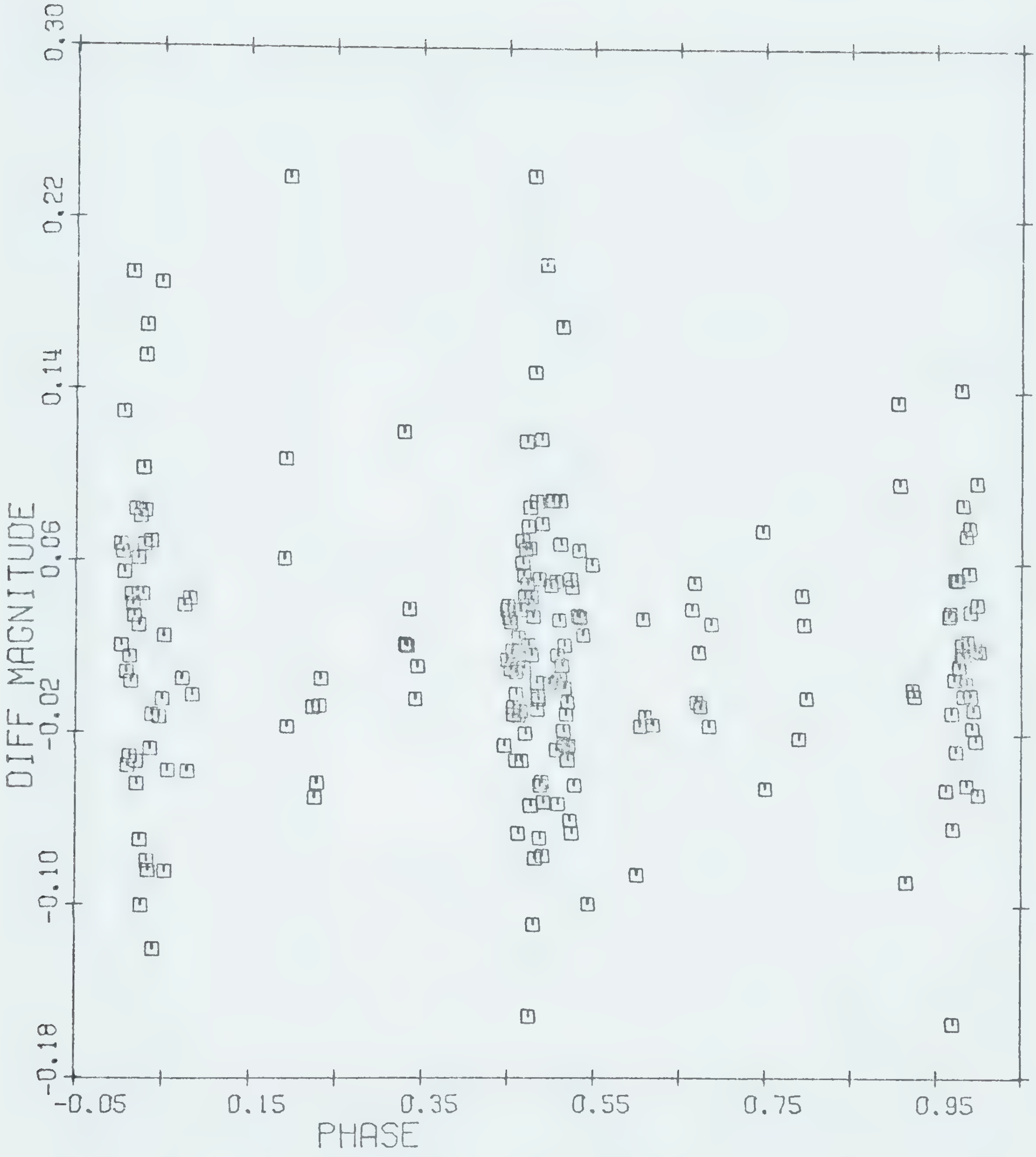
4.2 The Observations of the Comparison Star and Check Star

As a check on the constancy of the emitted radiation from the comparison star (HD 204426, $m_v = 6.90$, GO), measurements were made periodically on a third star (HD 204408, $M_v = 6.68$, AO). TABLE 4.3 lists the comparison - check star



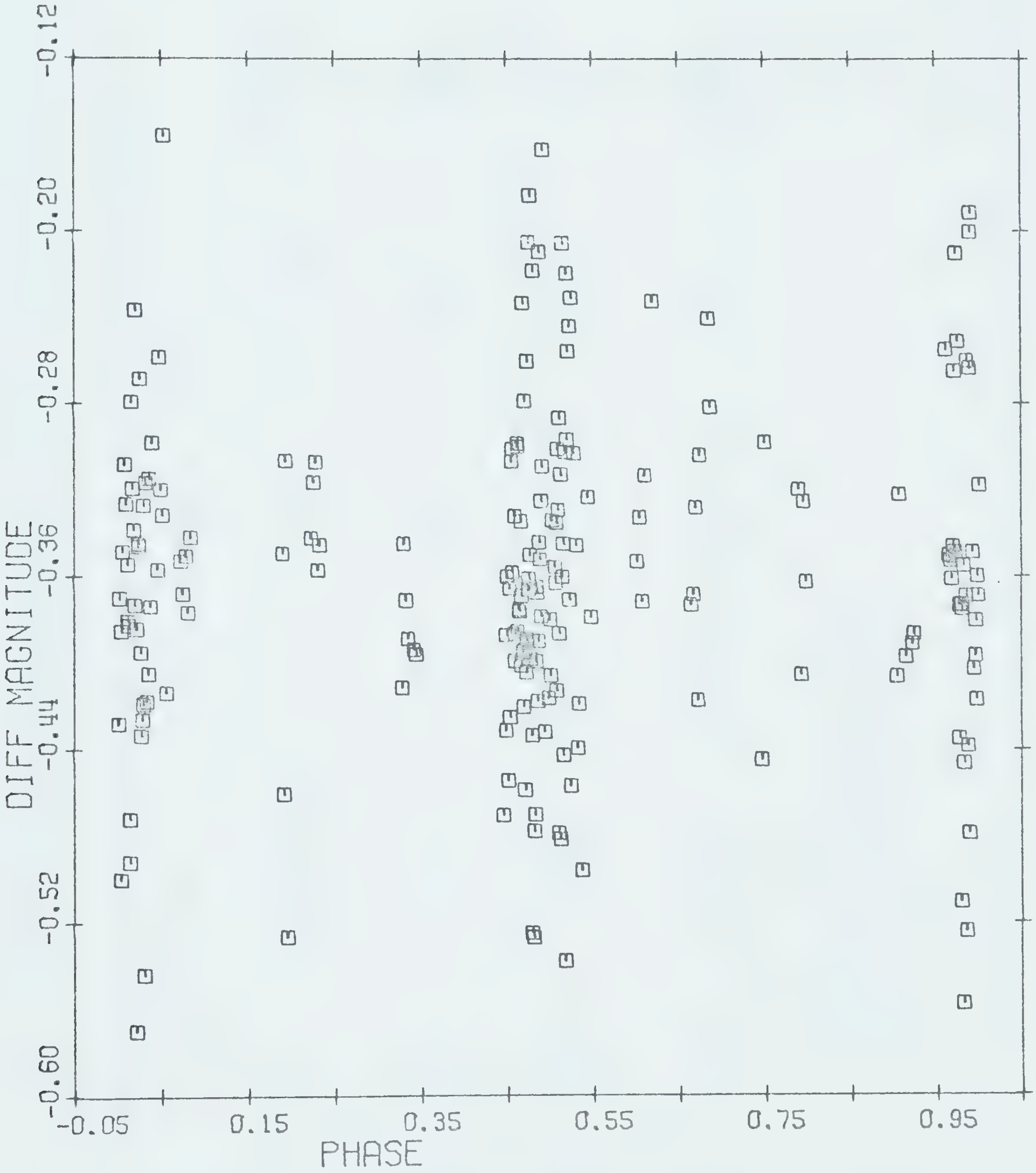
EI CEPHEI COLOUR INDEX B MINUS Y

FIGURE 4.13



EI CEPHEI COLOUR DIFFERENCE M1

FIGURE 4.14



EI CEPHEI COLOUR DIFFERENCE C1

FIGURE 4.15

TABLE 4.2

THE MEANS AND STANDARD DEVIATIONS OF THE COLOUR DIFFERENCES CORRESPONDING TO VARIOUS SECTIONS OF THE LIGHT CURVE

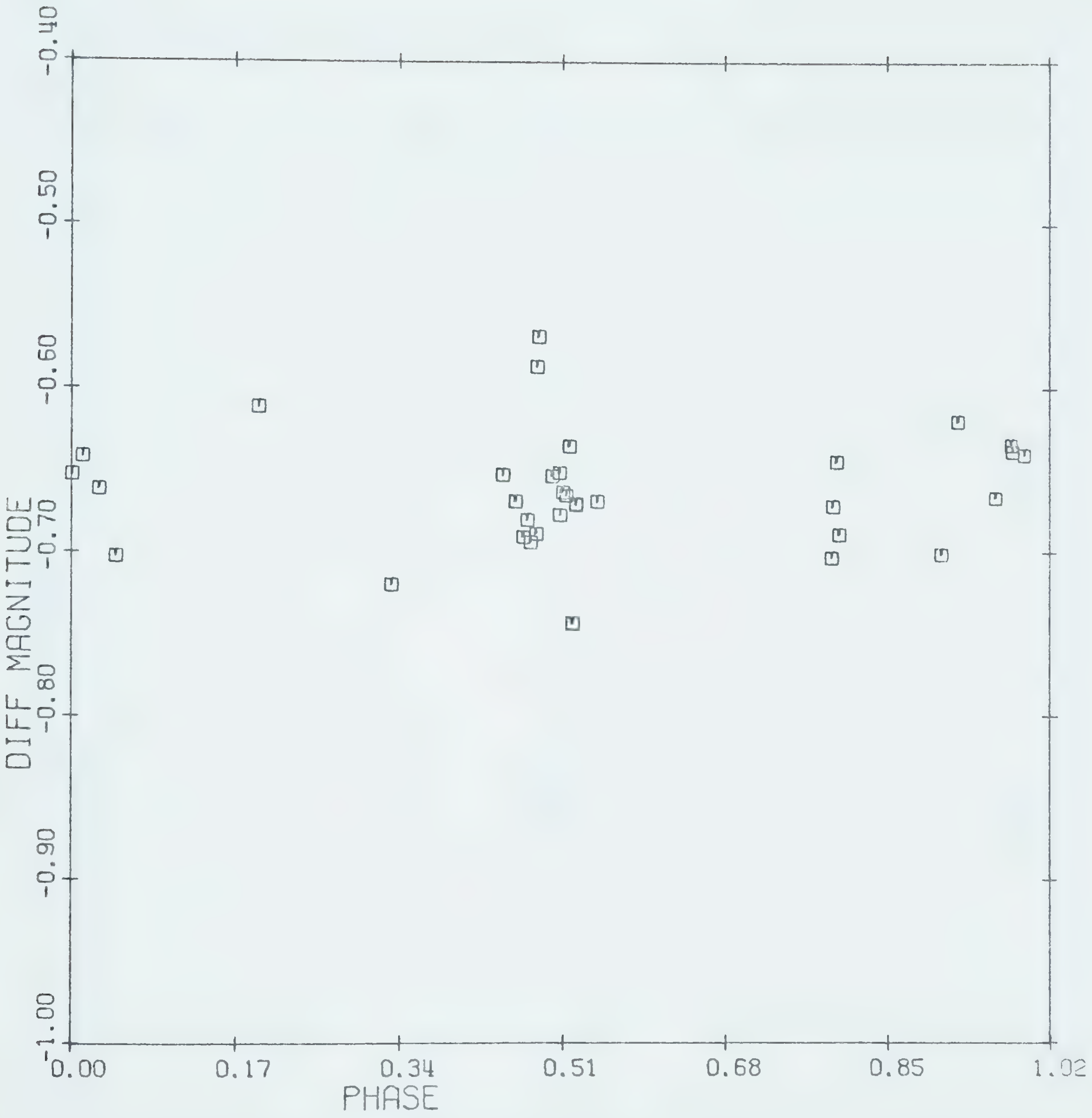
	P.A.*	OUTSIDE (P.A.-S.D.)	S.D.**	S.A.***	OUTSIDE (S.A.-P.D.)	P.D.****
(b-y)						
Mean	.153	.155	.155	.157	.153	.158
St. D.	.050	.033	.044	.031	.032	.041
m_1						
Mean	.022	.033	.019	.015	.011	.018
St. D.	.071	.072	.067	.053	.052	.060
c_1						
Mean	-.366	-.374	-.365	-.357	-.337	-.362
St. D.	.075	.057	.077	.080	.054	.082

* Primary Ascending Branch
** Secondary Descending Branch
*** Secondary Ascending Branch
**** Primary Descending Branch

TABLE 4.3

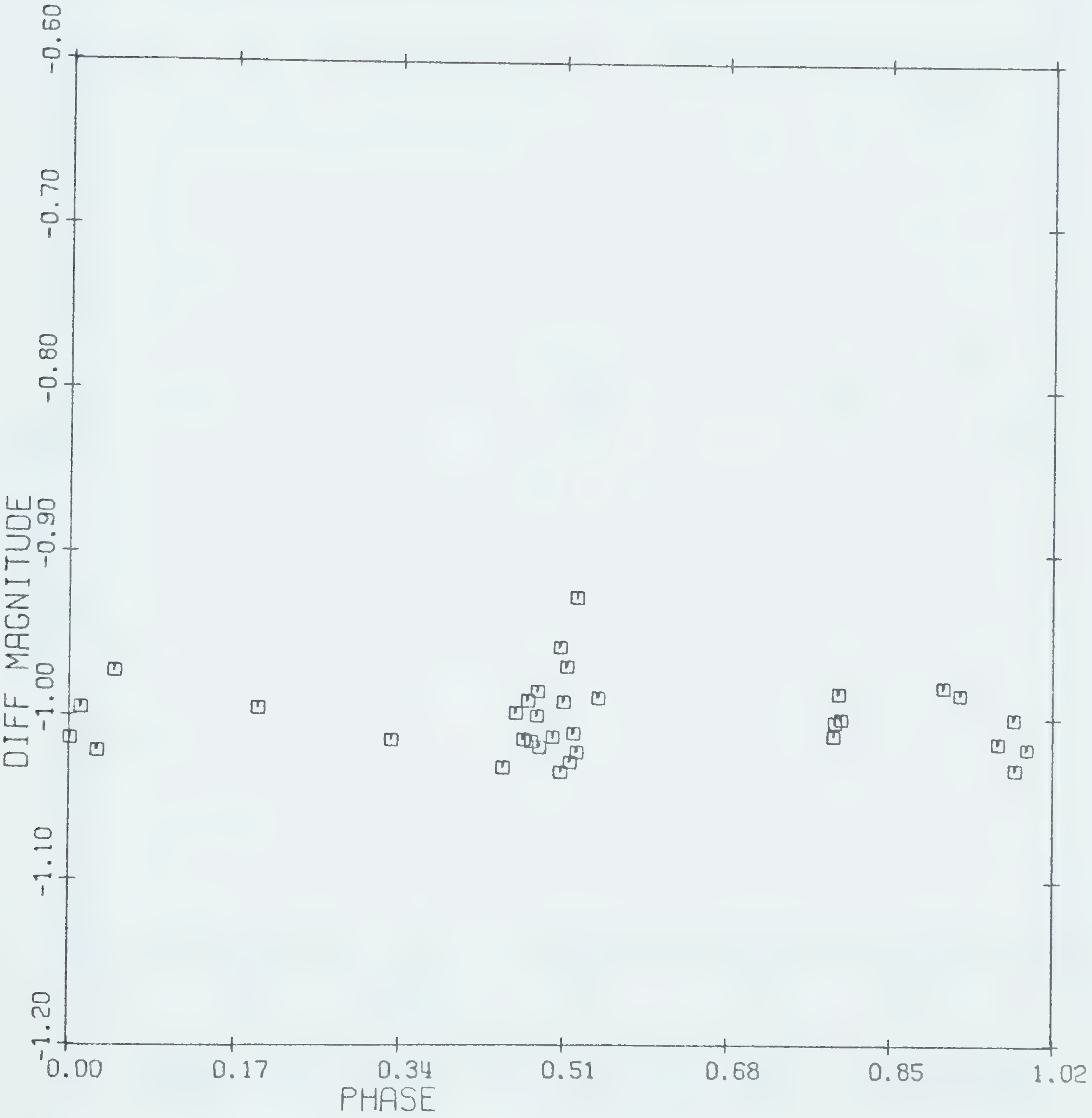
CHECK STAR DATA

PHASE	u	v	b	y
.0010	-0.652	-1.014	-0.623	-0.273
.0121	-0.641	-0.995	-0.587	-0.280
.0287	-0.661	-1.022	-0.628	-0.268
.0464	-0.702	-0.972	-0.608	-0.280
.1938	-0.611	-0.995	-0.655	-0.295
.3314	-0.720	-1.014	-0.619	-0.277
.4475	-0.653	-1.031	-0.621	-0.284
.4609	-0.669	-0.998	-0.608	-0.266
.4688	-0.691	-1.014	-0.636	-0.300
.4731	-0.680	-0.990	-0.622	-0.314
.4763	-0.694	-1.015	-0.622	-0.343
.4824	-0.689	-0.999	-0.640	-0.330
.4833	-0.587	-0.984	-0.588	-0.228
.4849	-0.568	-1.018	-0.615	-0.225
.4989	-0.653	-1.012	-0.616	-0.299
.5065	-0.651	-0.957	-0.610	-0.280
.5072	-0.677	-1.034	-0.649	-0.285
.5100	-0.663	-0.991	-0.611	-0.271
.5135	-0.665	-0.969	-0.634	-0.287
.5167	-0.635	-1.028	-0.647	-0.262
.5201	-0.743	-1.010	-0.628	-0.266
.5235	-0.670	-1.022	-0.626	-0.289
.5236	-0.671	-0.927	-0.619	-0.257
.5461	-0.669	-0.988	-0.632	-0.251
.7915	-0.703	-1.011	-0.632	-0.262
.7926	-0.672	-1.003	-0.646	-0.274
.7965	-0.645	-0.985	-0.638	-0.336
.7989	-0.689	-1.001	-0.628	-0.275
.9058	-0.701	-0.981	-0.606	-0.246
.9231	-0.620	-0.986	-0.639	-0.241
.9626	-0.667	-1.015	-0.665	-0.300
.9791	-0.635	-1.000	-0.619	-0.286
.9803	-0.638	-1.031	-0.659	-0.287
.9926	-0.641	-1.019	-0.613	-0.230



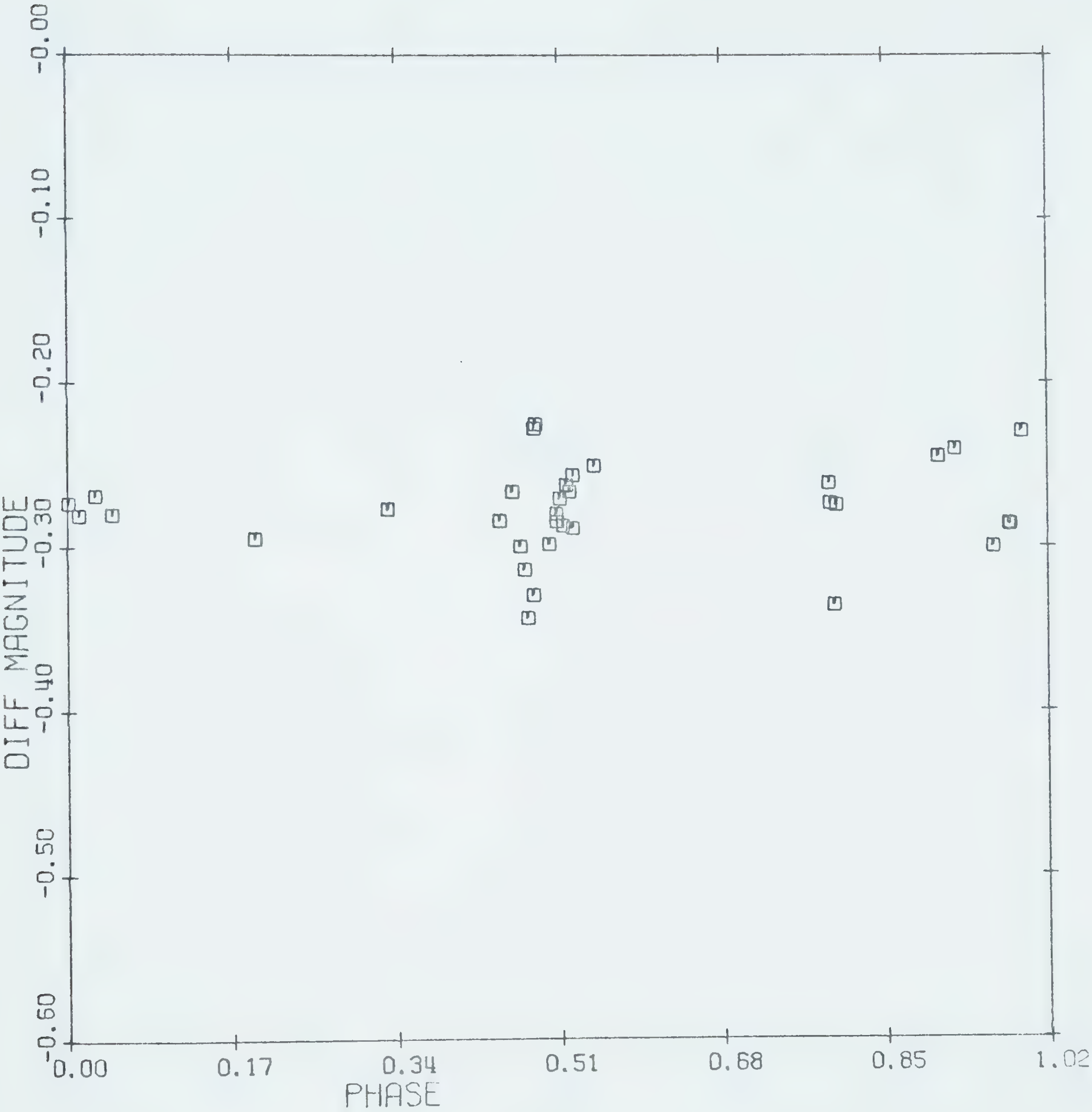
CHECK STAR MINUS COMPARISON STAR U-FILTER

FIGURE 4.16



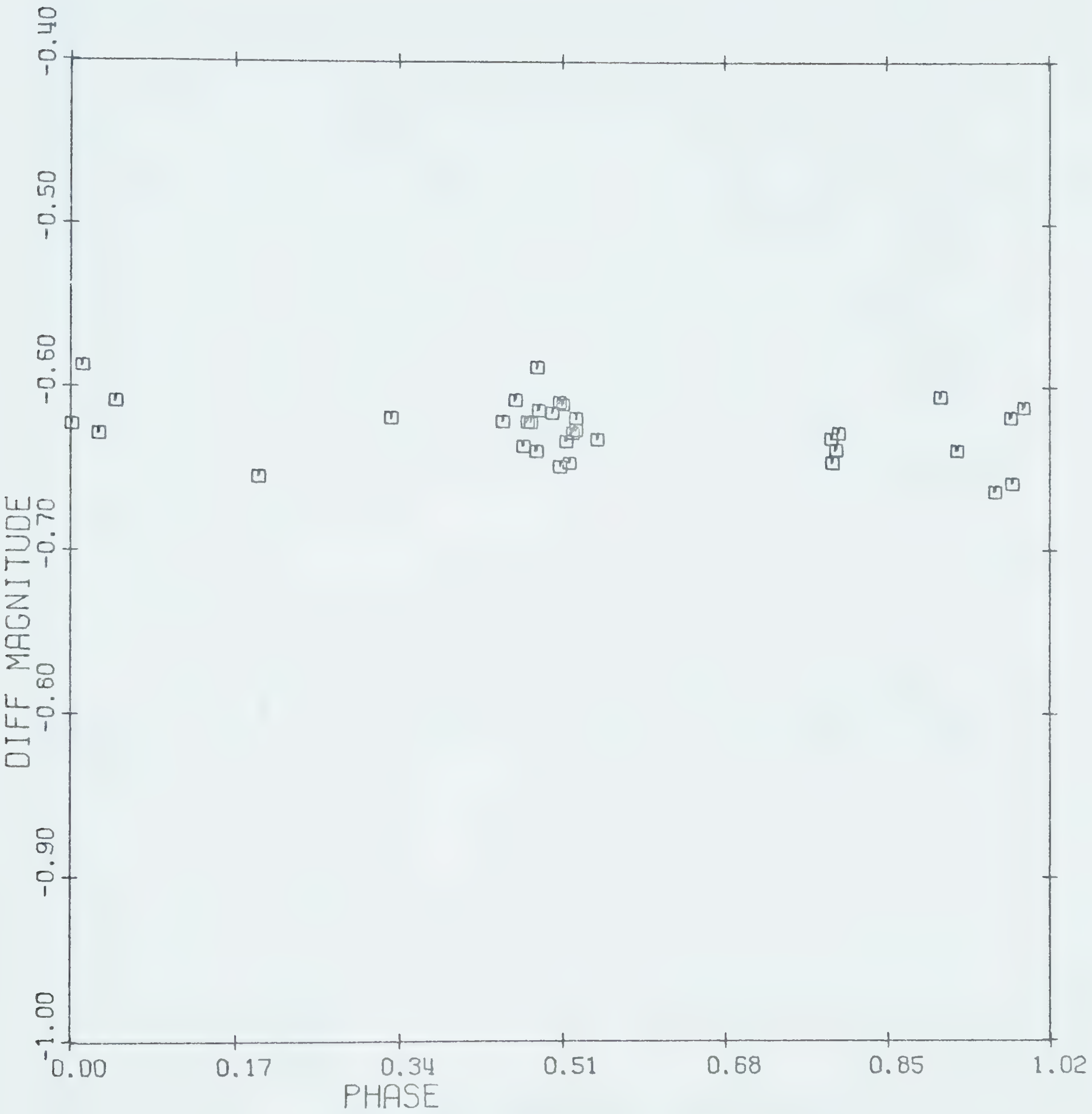
CHECK STAR MINUS COMPARISON STAR V-FILTER

FIGURE 4.17



CHECK STAR MINUS COMPARISON STAR Y-FILTER

FIGURE 4.18



CHECK STAR MINUS COMPARISON STAR   B-FILTER

FIGURE 4.19

data obtained during the course of investigation of EI CEP. The accompanying graphs portray the magnitude difference against the corresponding phase of EI CEP at the time of the measurement of the check star. All graphs bear a similar degree of scatter as found in the corresponding light curves. This fact plus the apparent increase in scatter during eclipse found for the colour indices lends more credence to the hypothesis that the distortions in the light curves are a result of the observational uncertainty rather than of any physical anomalies.

4.3 The Observation of Standard Stars

Measurements of the intensity of several standard stars were made on September 18, 1972, and October 4, 1972 from the following list:

(1) 88 δPEG
(2) 6 βARI
(3) 10 σTAU
(4) 8 δTAU
(5) π^3 ORI.

Upon reduction, it was found however, that the sagging effect in the photometer-telescope coupling had rendered the measurements unsuitable for accurate analysis. Different results were found for observations on opposite sides of the

zenith. Another set of data was taken in 1973 using the two stars δ Lyrae and i Aquilae. From the data obtained on δ Lyrae, it was possible to compute the extinction coefficients. The other star was observed too low on the horizon to make use of the corresponding measurements. The extinction coefficients calculated are

$K = .11\ (.150)$

$K_1 = .08\ (.068)$

$K_2 = .11\ (.187)$

$K_3 = .08\ (.053)$

The values in brackets are mean values measured at Kitt Peak by Crawford and Barnes.

Insufficient data made it impossible to calculate values of the transformation coefficients and therefore the measurements were left in the natural system during the analysis for the orbital elements. Due to the problems posed by the observations obtained from EI CEP, it was decided to maximize the observing time spent on the variable and comparison star so no further measurements of standard stars were made.

CHAPTER 5

ANALYSIS OF THE LIGHT CURVES AND CONCLUDING REMARKS

5.1 THE COMPUTER PROGRAM EVAL

The basis for the method of Kitamura, discussed in Chapter 3, is the evaluation of the incomplete Fourier transforms from which are computed the characteristic functions for that particular system. The evaluation of these transforms, requiring a numerical integration, was accomplished by means of the computer program EVAL written in FORTRAN for use with the IBM 36067. The first task required of EVAL was to convert the observations from differential magnitudes to luminosity units by inverting the equality $m = -2.5 \log \ell'$. The values of the transforms could then be calculated for each branch by means of a trapezoidal rule $^{(15)}$ of the form:

$$Sn' = \tfrac{1}{2} \sum_{i=1}^{m} \left[\ell' (\theta_i) \sin n\,\theta_i + \ell' (\theta_{i+1}) \sin n\theta_{i+1} \right] * \left[\theta_{i+1} - \theta_i \right] \qquad n = 1,2, \ldots$$

and

$$Cn' = \tfrac{1}{2} \sum_{i=1}^{m} \left[\ell' (\theta_i) \cos n\,\theta_i + \ell' (\theta_{i+1}) \cos n\,\theta_{i+1} \right] * \left[\theta_{i+1} - \theta_i \right] \qquad n = 1,2, \ldots$$

To ensure the best accuracy possible in the integration, the program also calculated the corresponding end corrections; viz.

$$\Delta C'_n = \frac{1}{12} h^2 n\ell'(\phi)\sin n\phi$$

$$\Delta S'_n = \frac{-1}{12} h^2 \left[n\ell'(\phi)\cos n\phi - n\ell'(0) \right]$$

Where h is the average interval of the phases of the observed points and ϕ is an arbitrary phase angle outside eclipse, while $\ell'(0)$ is the value of the luminosity of the closest point near mid-minimum. Once the above summations had been carried out, it was necessary to obtain a value that would normalize the light curve to bring it into line with the theoretical values of the light curve obtained from the α - functions which were used to calculate theoretical values of the characteristic functions. The normalizing factor for the light curves is obtained from the identity;

$$K = \frac{\phi_1 - \phi_2}{C_o(\phi_1) - C_o'(\phi_2)}$$

The quantities ϕ_1 and ϕ_2 represent two different phase angles outside eclipse while the denominator incorporates values of the incomplete Fourier transforms computed from the observed luminosities with n = 0. A normalizing factor was calculated for each branch of both minima, with the final value being their arithmetic mean. EVAL next computed

$$L{,}S_n = \frac{1 - \cos n\phi}{n} - \frac{1 - \cos n\varepsilon\lambda}{n} - KS'_n \quad n = 1,2,...$$

$$= 0$$

and

$$L{,}C_n = \frac{\sin n\phi}{n} - \frac{\sin n\varepsilon\lambda}{n} - KC'_n \quad n = 1,2,....$$

$$= \phi - \varepsilon\lambda - KC'_o$$

where λ is the normalized value of the depth of minimum estimated from the light curves and ε represents the phase angle of the point nearest to midminimum. If a value of phase angle occurs at exactly mid-eclipse then EVAL set ε automatically to zero. It is from the above values of $L{,}S_n$ and $L{,}C_n$ that the program finally calculates the characteristic functions,

$$F = \frac{L{,}S_n}{L{,}C_n} \qquad n = 1,2$$

$$E = \frac{L{,}C_o}{1 - \lambda}$$

Also evaluated is the ratio $(F_1/F_2)_{obs}$, which is a slowly varying function of the geometrical elements that becomes almost invariant in some ranges, and is a convenient comparison against (F_1/F_2) theo for detecting satisfactory sets of geometrical elements. The final task carried out by EVAL was to print out tabulated values of the incomplete transforms, characteristic functions, and the normalized values of the luminosity.

5.2 THE RESULTS OBTAINED BY USE OF EVAL

Before final reduction, the data were checked for measurements corresponding to the same phase. If found, the values of the differential magnitudes were averaged. This re-examination reduced the number of data points to 204. The output from EVAL obtained from the observations on EI CEPHEI are listed as follows:

TABLE 5.1	u-filter
TABLE 5.2	v-filter
TABLE 5.3	b-filter
TABLE 5.4	y-filter

The column headed with the letter ,K, is the normalizing factor for that particular branch. The next table illustrates the values of the estimated depths of minimum with their normalized values. The list of the normalized luminosities are numerically presented in TABLE 5.6 and graphically displayed by FIGS. 5.1, 5.2, 5.3, 5.4.

5.3 CONCLUSIONS CONCERNING EI CEP

The values of the characteristic functions for all four filters display a significant difference between branches. A comparison between minima reveals for the u.b.y filters that in all instances $E_{sec} < E_{prim}$,

TABLE 5.1

CALC OF INCOMPLETE TRANSFORMS BY THE TRAPAZOIDAL RULE u-FILTER

Minima	C0(1)	C0(2)	C1	C2	S1	S2
Primary Min.						
Asc.	0.162567	0.288669	0.274660	0.234955	0.078070	0.145353
Desc.	0.117546	0.288568	0.273712	0.231987	0.078817	0.145414
Sec. Min.						
Asc.	0.073574	0.158069	0.155460	0.147780	0.025232	0.049275
Desc.	0.109784	0.177561	0.173968	0.163429	0.031429	0.061065

CALC. OF CHARACTERISTIC FUNCTIONS E,F1,F2

Minima	K	D=L1C0	F1	F2	F1/F2	E
Primary Min.						
Asc.	1.709029	0.029907	0.034095	0.074973	0.454765	0.085111
Desc.	1.719376	0.036806	0.095078	0.228379	0.416316	0.104743
Sec. Min.						
Asc.	1.762344	0.030272	0.068470	0.137682	0.497303	0.088420
Desc.	1.696463	0.028952	0.042637	0.086138	0.494983	0.084563

The Normalization Factor is 1.721803

TABLE 5.2

CALC. OF INCOMPLETE TRANSFORMS BY THE TRAPAZOIDAL RULE v-FILTER

Minima	CO(1)	CO(2)	C1	C2	S1	S2
Primary Min.						
Asc.	0.193145	0.341438	0.324917	0.278099	0.092110	0.171493
Desc.	0.139314	0.341422	0.323515	0.273257	0.094085	0.173344
Sec. Min.						
Asc.	0.086497	0.189489	0.186320	0.176993	0.030496	0.059547
Desc.	0.131345	0.212458	0.208149	0.195511	0.037636	0.073120

CALC. OF CHARACTERISTIC FUNCTIONS E,F1,F2

Minima	K	D=L1CO	F1	F2	F1/F2	E
Primary Min.						
Asc.	1.453300	0.034273	0.074409	0.151634	0.490712	0.094836
Desc.	1.454917	0.041032	0.083726	0.207156	0.404172	0.113539
Sec. Min.						
Asc.	1.445836	0.029019	0.051943	0.104794	0.495663	0.086003
Desc.	1.417549	0.028125	0.037050	0.075429	0.491192	0.083351

The Normalization Factor is 1.442900

TABLE 5.3

CALC. OF INCOMPLETE TRANSFORMS BY THE TRAPAZOIDAL RULE b-FILTER

Minima	C0(1)	C0(2)	C1	C2	S1	S2
Primary Min.						
Asc.	0.164851	0.290369	0.276328	0.236539	0.078321	0.145824
Desc.	0.119032	0.294349	0.278967	0.235784	0.081093	0.149482
Sec. Min.						
Asc.	0.073891	0.162716	0.159988	0.151958	0.026242	0.051239
Desc.	0.111291	0.180102	0.176458	0.165770	0.031881	0.061944

CALC. OF CHARACTERISTIC FUNCTIONS E,F1,F2

Minima	K	D=L1C0	F1	F2	F1/F2	E
Primary Min.						
Asc.	1.716995	0.037538	0.092505	0.187002	0.494676	0.102000
Desc.	1.677253	0.037576	0.067168	0.177990	0.377369	0.102103
Sec. Min.						
Asc.	1.676436	0.028187	0.045443	0.091956	0.494182	0.084097
Desc.	1.670957	0.031135	0.051999	0.104491	0.497637	0.092829

The Normalization Factor is 1.685410

TABLE 5.4

CALC. OF INCOMPLETE TRANSFORMS BY THE TRAPAZOIDAL RULE y-FILTER

Minima	C0(1)	C0(2)	C1	C2	S1	S2
Primary Min.						
Asc.	0.142362	0.247671	0.235765	0.202032	0.066429	0.123670
Desc.	0.103196	0.251752	0.238742	0.202205	0.068937	0.127163
Sec. Min.						
Asc.	0.064390	0.140163	0.137846	0.131022	0.022395	0.043737
Desc.	0.097483	0.156156	0.153032	0.143869	0.027452	0.053351

CALC. OF CHARACTERISTIC FUNCTIONS E,F1,F2

Minima	K	D=L1C0	F1	F2	F1/F2	E
Primary Min.						
Asc.	2.046472	0.034639	0.099211	0.201961	0.491239	0.097084
Desc.	1.979398	0.033260	0.065499	0.177537	0.368929	0.093220
Sec. Min.						
Asc.	1.965207	0.023815	0.041716	0.084480	0.493795	0.075854
Desc.	1.959729	0.024304	0.032165	0.065123	0.493917	0.077412

The Normalization Factor is 1.987701

TABLE 5.5

DEPTH OF MINIMA

	ESTIMATED (MAGNITUDE)		NORMALIZED (LUMINOSITY)	
	PRIMARY	SECONDARY	PRIMARY	SECONDARY
u	-1.060	-1.045	.649	.658
v	-0.885	-0.845	.639	.663
b	-1.065	-1.010	.632	.665
y	-1.225	-1.155	.643	.686

TABLE 5.6

NORMALIZED VALUES OF LUMINOSITY

PHASE	u_{norm}	v_{norm}	b_{norm}	y_{norm}
.0009	0.653	0.662	0.631	0.646
.0020	0.647	0.648	0.641	0.654
.0034	0.635	0.679	0.639	0.644
.0042	0.721	0.698	0.661	0.711
.0053	0.677	0.660	0.651	0.681
.0078	0.750	0.725	0.735	0.759
.0096	0.713	0.709	0.728	0.730
.0114	0.738	0.747	0.760	0.792
.0117	0.805	0.820	0.820	0.805
.0128	0.745	0.772	0.784	0.807
.0138	0.802	0.847	0.807	0.809
.0142	0.793	0.826	0.762	0.849
.0155	0.872	0.818	0.828	0.877
.0169	0.908	0.871	0.868	0.901
.0183	0.922	0.885	0.866	0.925
.0194	0.847	0.883	0.910	0.918
.0198	0.930	0.871	0.914	0.931
.0217	0.791	0.942	0.926	0.973
.0220	0.962	0.960	0.936	0.947
.0240	0.971	0.949	0.942	0.947
.0254	0.985	0.926	0.947	0.892
.0263	0.944	0.974	0.974	1.024
.0269	0.926	0.940	0.892	0.939
.0284	0.993	1.000	0.949	0.967
.0295	1.051	1.048	0.992	1.023
.0297	1.042	1.007	1.002	1.162
.0304	0.956	1.032	0.940	1.012
.0319	1.021	1.014	1.049	1.018
.0333	0.978	1.051	1.073	1.023
.0353	0.942	0.988	0.996	0.988
.0359	1.012	0.977	0.985	1.068

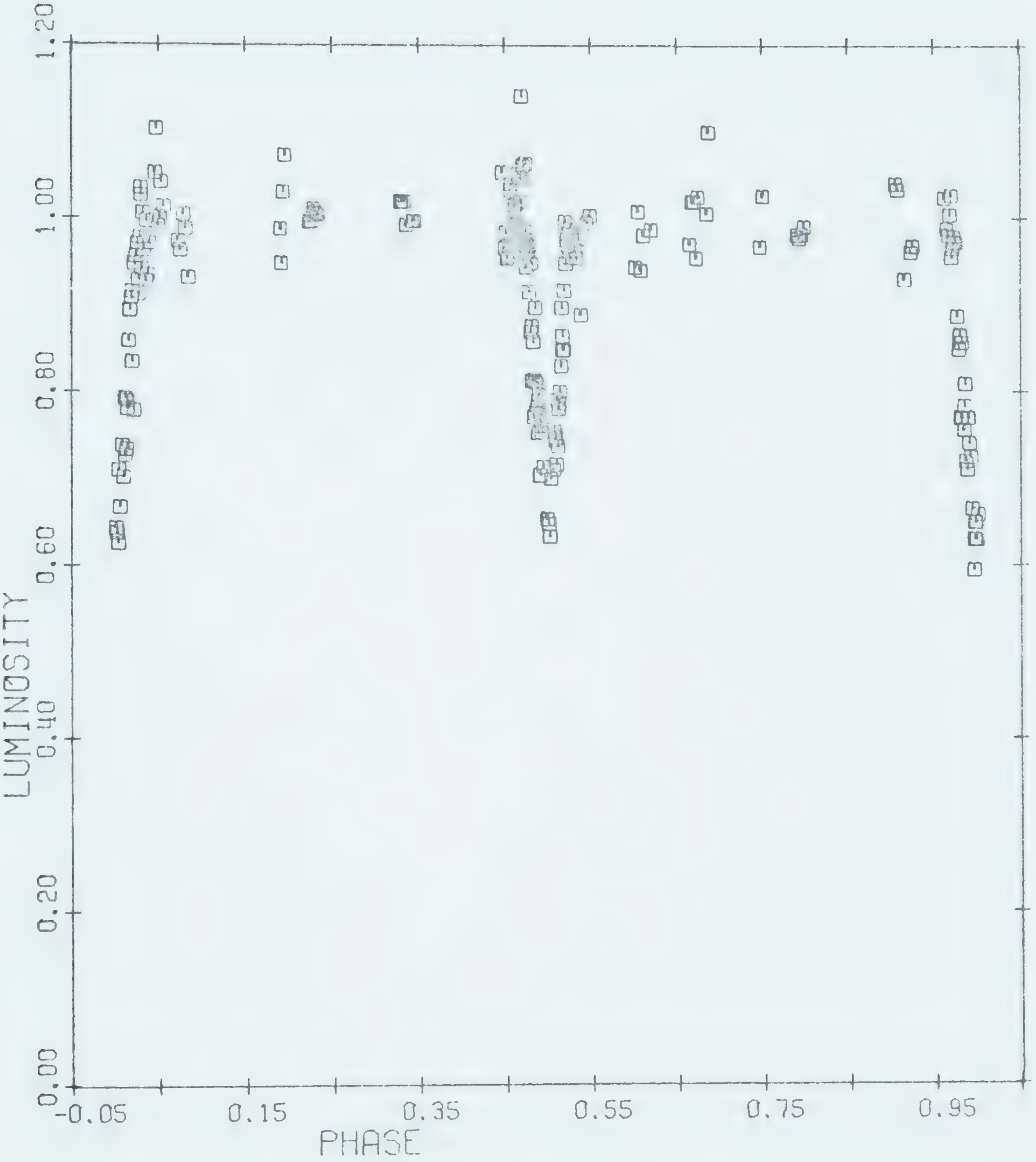
.0376	0.949	0.977	0.994	1.011
.0396	0.986	0.986	1.044	1.000
.0459	1.069	1.049	1.034	1.017
.0477	1.120	0.972	0.926	1.062
.0495	1.006	0.972	0.974	0.983
.0514	1.017	0.986	0.983	1.013
.0532	1.058	0.924	0.974	0.960
.0562	1.029	1.033	0.987	0.920
.0724	0.988	0.978	0.976	0.987
.0748	0.977	0.983	0.982	1.026
.0785	1.020	1.002	0.994	0.962
.0808	1.003	1.003	0.989	1.023
.0838	0.945	0.944	0.957	0.980
.1904	1.003	0.996	1.001	1.073
.1917	0.962	1.008	0.965	1.029
.1934	1.046	1.005	1.014	1.017
.1954	1.089	1.086	0.929	1.000
.2233	1.011	0.994	0.993	0.994
.2258	1.012	0.997	1.024	1.014
.2283	1.026	0.987	0.997	0.977
.2309	1.022	1.011	1.004	0.999
.2328	1.020	1.006	1.005	1.021
.3290	1.035	1.033	0.983	1.057
.3307	1.035	1.017	1.014	1.041
.3331	1.035	1.020	0.995	0.999
.3354	1.007	1.004	0.974	0.989
.3423	1.012	1.040	1.035	1.037
.3451	1.012	1.042	1.039	1.057
.4468	1.070	1.106	1.034	0.953
.4495	0.978	0.996	0.969	0.975
.4503	0.998	0.992	0.985	1.024
.4519	0.968	1.022	0.988	0.996
.4536	0.971	0.978	0.981	1.022
.4540	0.984	1.032	1.020	1.028
.4553	1.049	1.001	1.004	1.033

.4566	0.984	0.994	1.005	1.019
.4571	1.056	1.008	1.015	1.020
.4593	1.012	0.997	1.008	1.034
.4594	0.994	1.002	0.986	0.979
.4602	1.031	1.055	1.042	1.009
.4624	1.032	1.006	1.036	1.068
.4625	1.008	1.023	1.016	1.041
.4626	1.011	0.989	1.025	1.008
.4644	1.068	1.057	1.033	1.055
.4651	1.001	1.016	1.016	1.017
.4666	0.976	0.972	0.992	0.992
.4670	1.035	1.023	1.005	1.052
.4672	1.063	1.049	0.997	1.019
.4685	0.986	1.004	0.991	1.037
.4686	1.159	1.019	1.008	1.017
.4694	0.988	1.024	1.006	1.038
.4703	1.077	1.009	1.020	1.107
.4706	0.991	1.029	1.037	1.037
.4718	0.991	1.032	0.982	1.051
.4725	0.984	1.017	1.009	1.058
.4729	1.000	1.006	0.985	1.011
.4736	1.079	0.988	0.992	1.078
.4739	0.958	0.991	0.999	1.036
.4748	0.976	0.956	0.931	0.973
.4755	0.983	0.927	1.010	0.966
.4756	0.984	0.989	0.993	1.090
.4768	0.975	0.960	0.956	0.999
.4771	0.983	0.885	0.937	0.954
.4774	0.929	0.949	0.937	0.947
.4793	0.882	0.928	0.914	0.937
.4794	0.963	0.988	0.872	0.969
.4807	0.889	0.838	0.900	0.883
.4809	0.824	0.906	0.855	0.934
.4822	0.870	0.944	0.919	0.839
.4828	0.782	0.846	0.826	0.885

.4841	0.825	0.818	0.783	0.759
.4846	0.910	0.890	0.866	0.844
.4848	0.787	0.808	0.825	0.848
.4861	0.823	0.854	0.842	0.835
.4864	0.794	0.814	0.813	0.859
.4874	0.802	0.736	0.776	0.794
.4876	0.764	0.771	0.789	0.766
.4883	0.808	0.776	0.751	0.817
.4893	0.786	0.776	0.754	0.794
.4899	0.714	0.721	0.752	0.761
.4912	0.773	0.771	0.808	0.796
.4921	0.764	0.678	0.722	0.740
.4942	0.724	0.714	0.659	0.737
.4986	0.663	0.696	0.694	0.701
.5001	0.657	0.666	0.662	0.697
.5011	0.642	0.657	0.644	0.692
.5026	0.711	0.686	0.678	0.735
.5061	0.721	0.718	0.717	0.759
.5066	0.766	0.770	0.773	0.763
.5077	0.756	0.746	0.754	0.772
.5082	0.759	0.791	0.785	0.799
.5090	0.726	0.729	0.773	0.787
.5096	0.802	0.769	0.758	0.778
.5106	0.748	0.781	0.731	0.750
.5110	0.792	0.775	0.775	0.811
.5127	0.798	0.825	0.763	0.833
.5130	0.812	0.797	0.818	0.856
.5145	0.842	0.849	0.856	0.852
.5150	0.910	0.816	0.844	0.867
.5161	0.861	0.895	0.864	0.857
.5162	0.877	0.873	0.882	0.900
.5178	0.861	0.952	0.895	0.840
.5181	0.931	0.903	0.924	0.931
.5196	1.012	0.890	0.891	0.896
.5202	0.962	0.933	0.959	0.966

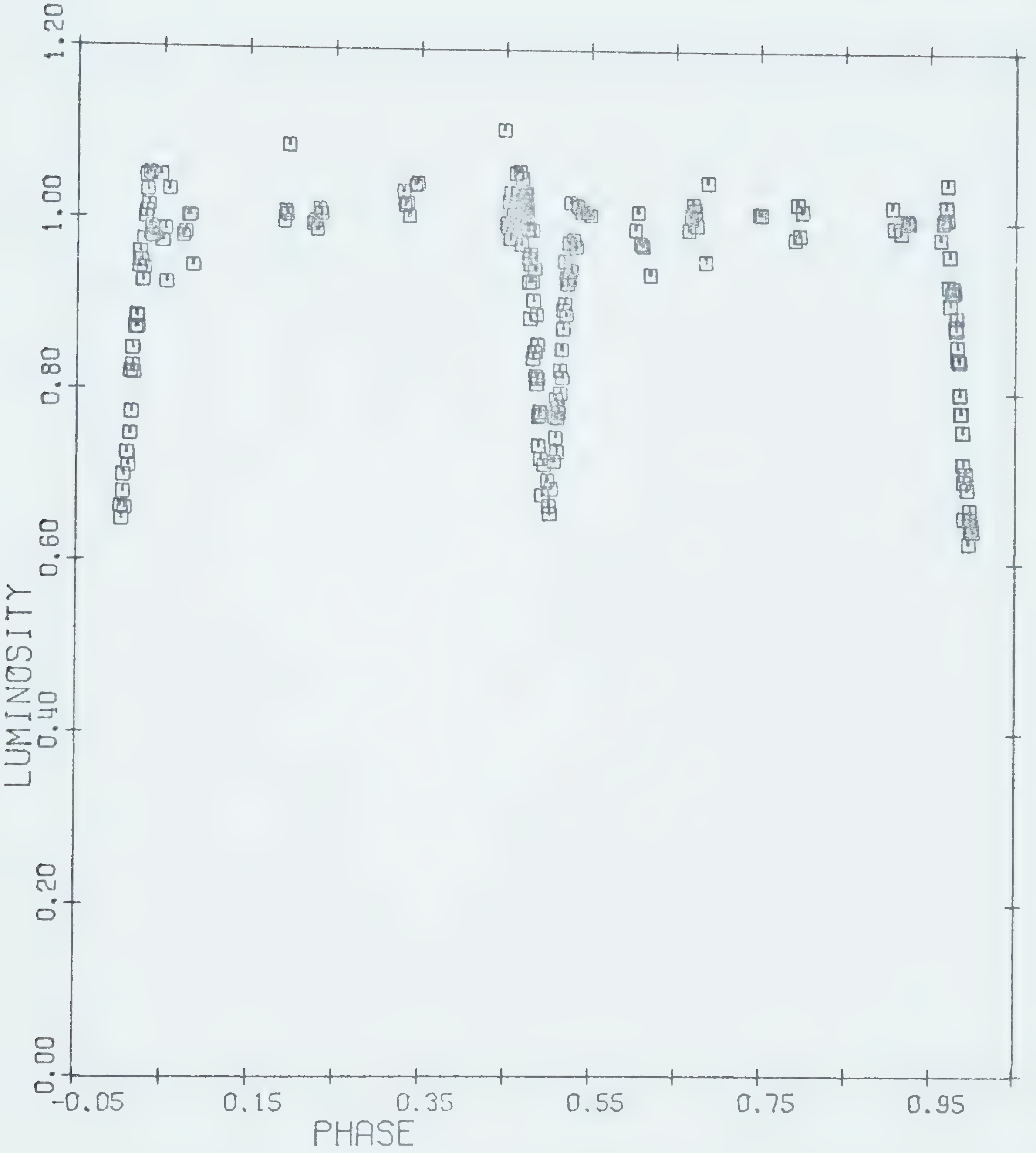
.5210	0.990	0.926	0.953	0.968
.5230	0.983	0.932	0.982	0.990
.5231	0.997	0.974	0.943	0.966
.5245	0.969	1.022	0.986	1.005
.5249	1.003	0.942	0.997	1.002
.5281	0.997	0.977	1.010	1.012
.5309	0.978	0.969	0.973	1.019
.5325	0.968	1.018	0.995	1.044
.5341	0.991	1.016	0.987	1.000
.5373	0.902	1.010	0.999	1.021
.5444	1.010	1.010	1.046	0.997
.5482	1.019	1.007	0.978	1.013
.6017	0.958	0.990	1.031	1.001
.6047	1.023	1.010	1.022	1.029
.6075	0.954	0.974	0.984	1.036
.6107	0.995	0.971	0.989	1.005
.6195	1.002	0.937	0.986	1.033
.6652	0.984	0.990	0.984	1.023
.6674	1.035	1.007	0.971	0.992
.6700	1.035	1.020	1.034	1.055
.6726	0.968	1.014	1.007	1.027
.6745	1.040	0.996	1.004	1.015
.6848	1.020	0.952	0.993	1.030
.6870	1.117	1.045	1.051	1.098
.7476	0.982	1.009	0.960	0.988
.7503	1.041	1.008	1.034	1.026
.7906	0.995	0.979	1.001	1.012
.7930	0.992	1.021	1.008	1.047
.7957	0.996	0.985	1.005	1.067
.7983	1.006	1.012	1.017	1.030
.9049	1.056	1.018	0.940	0.993
.9071	1.049	0.994	0.976	1.058
.9150	0.944	0.988	1.001	0.943
.9226	0.975	1.003	1.003	1.012

.9244	0.983	1.000	0.992	0.994
.9618	1.039	0.981	1.019	1.026
.9645	1.003	1.001	1.009	1.061
.9666	0.995	1.004	1.019	1.082
.9679	1.020	1.019	1.017	1.017
.9695	1.042	1.045	1.059	1.019
.9696	0.969	1.005	1.056	0.970
.9712	0.980	0.926	0.956	1.002
.9721	0.994	0.961	0.940	0.974
.9728	0.987	0.904	0.950	0.983
.9748	0.987	0.918	0.943	1.028
.9765	0.900	0.924	0.910	0.915
.9787	0.861	0.920	0.857	0.916
.9794	0.877	0.878	0.867	0.878
.9798	0.878	0.875	0.861	0.873
.9807	0.781	0.889	0.842	0.875
.9814	0.869	0.854	0.843	0.839
.9822	0.781	0.840	0.835	0.850
.9844	0.767	0.837	0.786	0.796
.9845	0.795	0.799	0.796	0.805
.9854	0.821	0.777	0.806	0.811
.9864	0.730	0.777	0.769	0.786
.9879	0.720	0.755	0.710	0.711
.9884	0.781	0.717	0.719	0.782
.9894	0.781	0.697	0.720	0.778
.9901	0.752	0.654	0.664	0.680
.9921	0.735	0.707	0.687	0.663
.9937	0.675	0.687	0.674	0.662
.9957	0.603	0.624	0.623	0.641
.9958	0.639	0.653	0.655	0.649
.9967	0.659	0.663	0.633	0.668
.9977	0.659	0.642	0.626	0.639
.9987	0.639	0.646	0.649	0.629
.9999	0.668	0.639	0.635	0.649



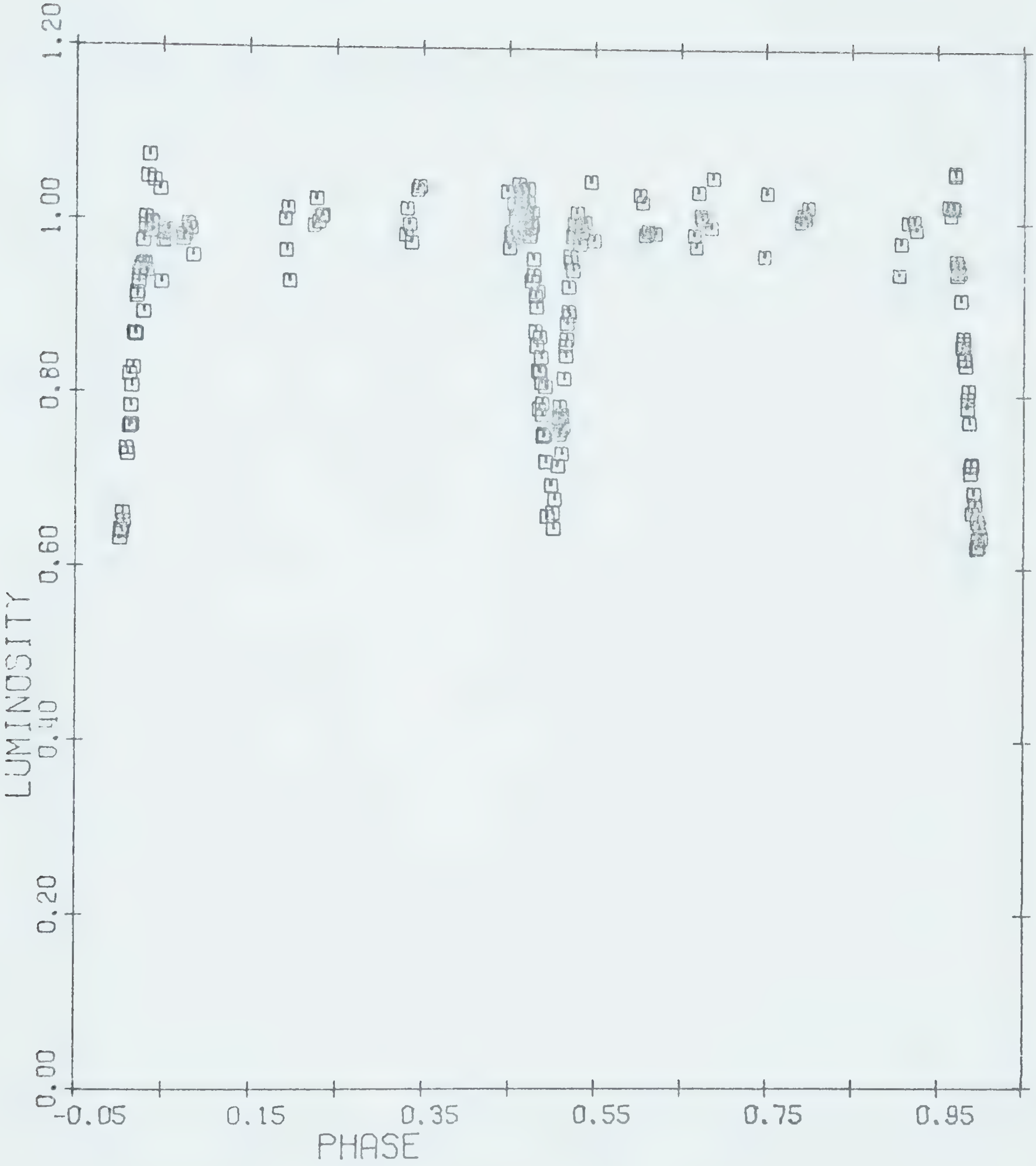
EI CEPHEI NORMALIZED LIGHT CURVE U-FILTER

FIGURE 5.1



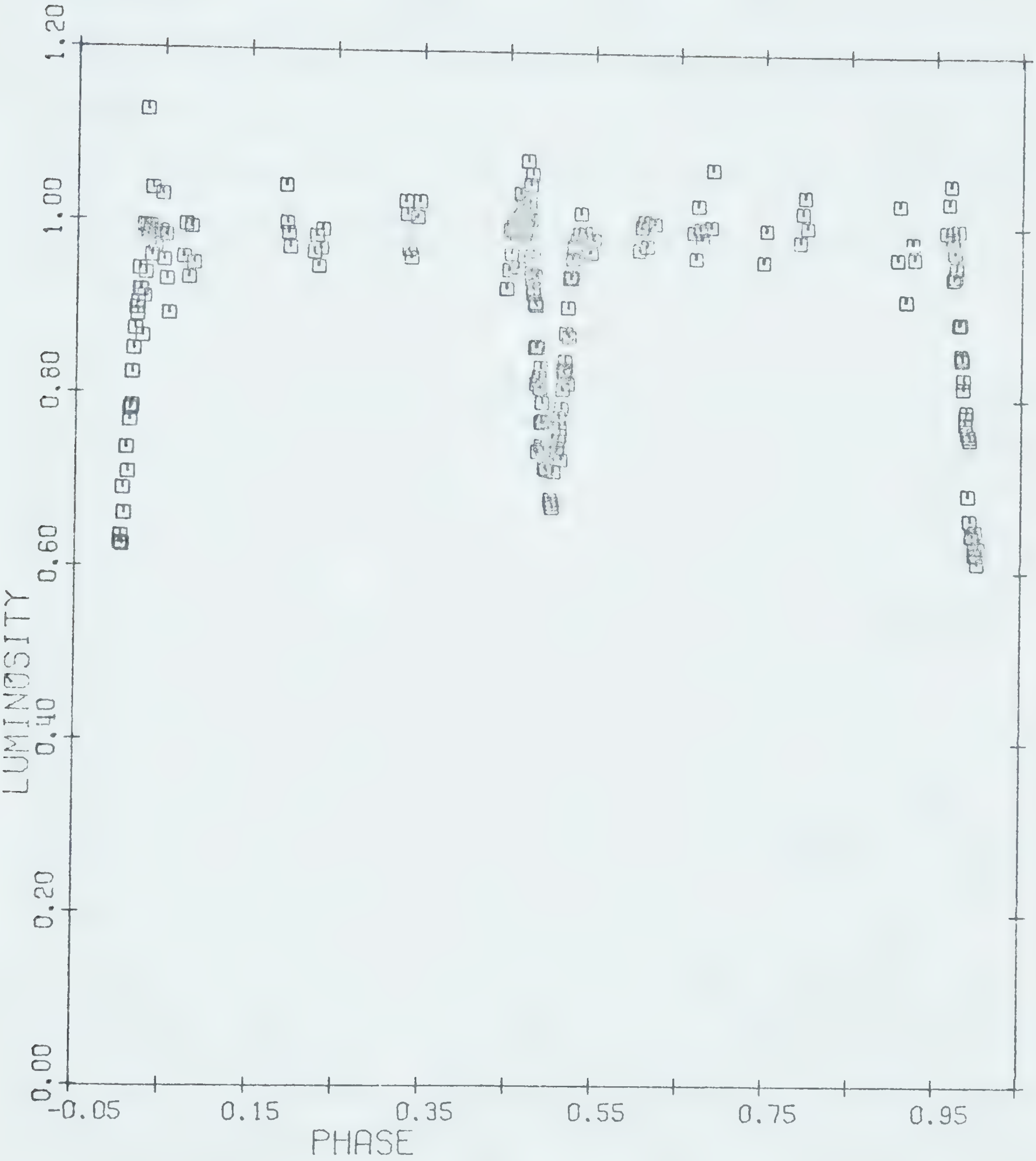
EI CEPHEI NORMALIZED LIGHT CURVE   V-FILTER

FIGURE 5.2



EI CEPHEI NORMALIZED LIGHT CURVE B-FILTER

FIGURE 5.3



EI CEPHEI NORMALIZED LIGHT CURVE Y-FILTER

FIGURE 5.4

F_1sec < F_1prim, and F_2sec < F_2prim. The above inequalities hold for the u-filter if average values between branches are taken. This implies, using the relationships given by Kitamura namely $E^{tr} \leq E^{oc}$ and $F_n^{tr} \leq F_n^{oc}$, that the primary minimum of EI CEP is an occultation while the secondary is a transit. This result is consistent with the conclusions arrived at by Abrami.

The discrepancy between the characteristic functions of the ascending and descending limbs are such as to leave the possibility of a final solution to the light curves somewhat in doubt. While the values do agree to two decimal places, agreement to the third or fourth is required to use the theoretical tables of the characteristic functions successfully. This close agreement between values is especially required of the ratio $(F_1/F_2)_{obs}$ because of its slow variation with respect to the geometrical elements. The required agreement, however, is not achieved as the values obtained for the descending branch of the primary minimum are considerably lower than the other three for all the filters. Despite the amount of the discrepancy between branches, an attempt was made at a solution for the b-filter which displayed the closest agreement. An

extensive search through the TABLES failed to yield a reasonable set of theoretical values to match the observational values and therefore a complete analysis could not be realized.

5.4 CONCLUDING REMARKS

This initial test of a narrow passband filter system used in conjunction with the University of Alberta telescope has posed several problems. First, it appears that the employment of this system on a faint star introduces a large scatter. Although the light curve is discernable it is not of sufficient accuracy to allow for complete analysis using the method of Kitamura. A major question arises from whether or not the scatter is a result of simply using too faint a system and overextending the bounds of the equipment or rather an intrinsic property of a narrow band-small telescope union. It would seem an appropriate step to re-examine the uvby system under different conditions. This new study should involve an eclipsing binary of brighter magnitude, shorter period, and possibly with an existing accurate light curve. The need for the increased luminosity is to reduce the back ground effects of the sky by increasing the signal from the

photometer for all filters yet maintaining an operating voltage close to the value used on EI CEPHEI (1900V). The necessity of a shorter period is to ensure that each eclipse may be observed more fully on a single night and that more nights of observing are available when the star is in minimum. The concept of using an eclipsing binary with a previously measured light curve is to determine the overall effect of the scatter (if any is present) on the shapes of the minima.

If the scatter is still prevalent, then it may be necessary to take measurements of the comparison star before and after the variable to improve the accuracy. This process should be facilitated with the new mounting which is motor driven in both right ascension and declination. Constant monitoring of the sky conditions is essential so that it does not become an appreciable fraction of the voltages received from the stars being observed. During observations of EI CEP, these readings could be as high as thirty percent of the signal obtained in the u-filter. Consideration should be given also to the possible extended use of a pulse counter to determine if any gain in accuracy can be achieved over the digital voltmeter now in operation.

The measurement of a bright standard star prior to a night's observation may indicate the reproducibility of the equipment. If monitored throughout the night, it could give some indication of the operation of the photometer during the course of the night.

Although the light curves obtained were inadequate for a complete analysis, EI CEPHEI proved an interesting system to work with and in itself is worthy of a closer, more accurate study.

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APPENDIX I

In order to make the most effective use of EVAL a short program CORR should be executed first. The prime objectives of this program are to add the heliocentric corrections, calculate the colour differences and to order the data in a manner compatible with the input requirements of EVAL. Since this program together with EVAL was written primarily for EI CEP certain changes may be necessary to make these programs suitable for other variables. For instance to obtain the entire printed and punched output from CORR, the programmer will have to delete all the C's placed in the first column of some of the statements. These were used when the program was rerun and not all the output was essential. The input data for CORR is;

(1) the titles of the graphs of the light curves, (b-y), m_1, and c_1. For a detailed explanation, the reader is referred to the COMPUTING CENTRE publication entitled SYSTEMS SUBROUTINES in particular CGPL.

(2) the following card contains the constants required by the program;

  (a) RIGHT ASCENSION (ALFA) of the variable expressed in radians, F7.0,

  (b) DECLINATION (DELTA) of the variable expressed in radians, F7.0

(c) ANGLE of the ECLIPTIC (ECLIP) expressed in radians, F7.0

(d) PERIOD of the variable (PERIOD) F9.6,

(e) EPOCH OF PRIMARY CONJUNCTION (TO) expressed by the corresponding Julian date, F13.4.

(3) The next cards put the nightly data in memory. The first card of each night's observations contains,

(a) the Julian Day at the beginning of the observing night (XJDAY), F9.0,

(b) the number of observations made during that night (NO),I3,

(c) the normalized EARTH-SUN radius for the above Julian Day, (RAD), F9.0,

(d) the longitude of the EARTH in its orbit expressed in degrees (DS), F5.0, minutes (SM), F4.0, seconds (SS), F6.0.

The last two quantities are obtained from tables given in the NAUTICAL ALMANAC. The last cards list the reduced data from the digital voltmeter with,

(a) the time of each observation in hours (HR), F4.0, and minutes (MIN), F4.0,

(b) the differential magnitudes computed from the raw data in the order u(XU), v(XV), b(XB), y(XY), 4F7.0.

All the above format codes should be considered as flexible and easily altered to more suitable values. The values of the differential magnitudes for EI CEP were negative but for convenience were read in as positive and adjusted within CORR. Therefore for data which is intrinsically positive it will be necessary to remove the negative signs present in the statements that assign these values (XU, XV, XB,XY) to U(I), V(I), B(I), and Y(I).

The output data from CORR consists of the printed values of the PHASE, U(I), V(I), B(I), Y(I), (b-y), M_1, C_1, and I. The value of the total number of data points, MAX, together with the PHASE and the four magnitudes are punched on cards ready for use with EVAL. The cards are punched in increasing order of phase from .0000 to I.0000 with I representing a running number from 1 which corresponds to the phase nearest .0000. The use of I simplifies the determination of the limits of the summations used in the next program.

```
      REAL ALFA,DELTA,ECLIP,V(500),U(500),B(500),Y(500),BMY(500),
     1C1(500),M1(500),MIN,PHASE(500),PI/3.141593/,SISQF(500),
     1DUMMY(20)/20*4H    /,
     1T1(20),T2(20),T3(20),T4(20),T5(20),T6(20),T7(20),T8(20)
      REAL*8 XJDAY,TO,PERIOD,YY,T,PH,DPHASE,XMP,DFLOAT
      READ(5,1) T1
 1    FORMAT(20A4)
      READ(5,2) T2
 2    FORMAT(20A4)
      READ(5,3) T3
 3    FORMAT(20A4)
      READ(5,4) T4
 4    FORMAT(20A4)
      READ(5,5) T5
 5    FORMAT(20A4)
      READ(5,6) T6
 6    FORMAT(20A4)
      READ(5,7) T7
 7    FORMAT(20A4)
      READ(5,8) T8
 8    FORMAT(20A4)
      READ(5,111) ALFA,DELTA,ECLIP,PERIOD,TO
111   FORMAT(3F7.0,F9.6,F13.4)
      MAX=0
      I=0
901   READ(5,222,END=999) XJDAY,NO,RAD,SD,SM,SS
222   FORMAT(F9.0,I3,F8.0,F5.0,F4.0,F6.0)
      SUN=(SD+SM/60.+SS/3600.)*.0174533
      YY=HELIO(ALFA,DELTA,ECLIP,SUN,RAD)+XJDAY
      MAX=MAX+NO
      DO 100 J=1,NO
      I=I+1
      READ(5,333) HR,MIN,XU,XV,XB,XY
333   FORMAT(2F4.0,4F7.0)
      U(I)=-XU
      V(I)=-XV
```

```
      B(I)=-XB
      Y(I)=-XY
      BMY(I)=B(I)-Y(I)
      M1(I)=V(I)+Y(I)-2*B(I)
      C1(I)=U(I)+B(I)-2*V(I)
      T=(HR+MIN/60.)/24.+YY
      PH=(T-TO)/PERIOD
      RV=SNGL(PH)
      MP=IFIX(RV)
      XMP=DFLOAT(MP)
      DPHASE=PH-XMP
      PHASE(I)=SNGL(DPHASE)
  100 CONTINUE
      GO TO 901
  999 IFUDGE=MAX-1
C     WRITE(6,888) MAX
C888  FORMAT('THERE ARE  ',I3,'  NO. OF OBSERVATIONS')
C     WRITE(7,889) MAX
C889  FORMAT(I3)

      DO 200 J=1,IFUDGE
      ICOUNT=0
      NDEC=MAX-J
      DO 300 I=1,NDEC
      IF (PHASE(I).LT.PHASE(I+1)) GO TO 300
      TEMP=PHASE(I)
      PHASE(I)=PHASE(I+1)
      PHASE(I+1)=TEMP
      TEMP1=U(I)
      U(I)=U(I+1)
      U(I+1)=TEMP1
      TEMP2=V(I)
      V(I)=V(I+1)
      V(I+1)=TEMP2
      TEMP3=B(I)
      B(I)=B(I+1)
```

```
      B(I+1)=TEMP3
      TEMP4=Y(I)
      Y(I)=Y(I+1)
      Y(I+1)=TEMP4
      TEMP5=BMY(I)
      BMY(I)=BMY(I+1)
      BMY(I+1)=TEMP5
      TEMP6=M1(I)
      M1(I)=M1(I+1)
      M1(I+1)=TEMP6
      TEMP7=C1(I)
      C1(I)=C1(I+1)
      C1(I+1)=TEMP7
      ICOUNT=1
  300 CONTINUE
      IF(ICOUNT.EQ.0) GO TO 401
  200 CONTINUE
C     WRITE(6,445) DUMMY
C445  FORMAT('1',6X,'PHASE',7X,'U',10X,'V',10X,'B',10X,'Y',9X,'B-Y',9X
C    1,'M1',10X,'C1',9X,'I',20A4)
C401  DO 400 I=1,MAX
C     WRITE(6,444) PHASE(I),U(I),V(I),B(I),Y(I),BMY(I),M1(I),C1(I),I
C444  FORMAT('0',5X,F6.4,4X,F7.3,4X,F7.3,4X,F7.3,4X,F7.3,
C    14X,F7.3,4X,F8.3,4X,F8.3,4X,I3)
C     WRITE(7,777) PHASE(I),U(I),V(I),B(I),Y(I)
C777  FORMAT(F6.4,1X,F7.3,1X,F7.3,1X,F7.3,1X,F7.3)
C     SISQF(I)=(SIN(PHASE(I)*2.*PI))**2
C400  CONTINUE
C401  CALL CGPL(PHASE,U,U,MAX,1,1,1,1,1,-.05,.20,5.5,-1.300,.17,6.0,
C    1T1,6)
C     CALL CGPL(PHASE,V,V,MAX,1,1,1,1,1,-.05,.20,5.5,-1.300,.17,6.0,
C    1T2,6)
C     CALL CGPL(PHASE,B,B,MAX,1,1,1,1,1,-.05,.20,5.5,-1.300,.17,6.0,
C    1T3,6)
C     CALL CGPL(PHASE,Y,Y,MAX,1,1,1,1,1,-.05,.20,5.5,-1.300,.17,6.0,
C    1T4,6)
```

```
401  CALL CGPL(PHASE,BMY,BMY,MAX,1,1,1,1,1,-.05,.20,5.5,-.18,
    10.08,6.0,T5,6)
     CALL CGPL(PHASE,M1,M1,MAX,1,1,1,1,1,-.05,.20,5.5,-.18,

 10.08,6.0,T6,6)
  CALL CGPL(PHASF,C1,C1,MAX,1,1,1,1,1,-.05,.20,5.5,-.60,
 1.08,6.0,T7,6)
  CALL CGPL(PHASE,Y,Y,MAX,0,1,1,1,1,-.05,.20,5.5,-1.300,.17,6.0,
 1T4,6)
  STOP
  END



      FUNCTION HELIO(ALFA,DELTA,ECLIP,SUN,RAD)
      Q=COS(DELTA)*COS(ALFA)
      R=TAN(ECLIP)*SIN(DELTA)+COS(DFLTA)*SIN(ALFA)
      S=RAD*COS(SUN)
      TT=RAD*SIN(SUN)*COS(ECLIP)
      HELIO=-0.005770*(Q*S+R*TT)
      RETURN
      END
```

APPENDIX II

The input data for EVAL consist of,

(1) the titles of the graphs of the normalized light curves as explained in the preceding appendix,

(2) the estimated values of the depth of primary and secondary minima for each of the filters expressed in units of magnitude (ULMP, ULMS, VLMP, VLMS, BLMP, BLMS, YLMP, YLMS), 8F8.0,

(3) the following values are the integer numbers, I, of the various limits required for the integration;

  (a) IFI1, value of phase angle outside eclipse (1 for each branch), 14,

  (b) IFI2, a different phase angle further out side eclipse (1 for each branch) 14,

  (c) NEAR, the point on the branch being integrated closest to midminimum (1 for each branch), 14,

  (d) IEPS, the point nearest to mid-eclipse on any of the two branches forming a minimum (1 for each minimum), 14.

The values of NEAR and IEPS will be identical for one branch in each minimum. The order of the branches being summed

over is primary ascending, primary descending, secondary ascending, and secondary descending.

(4) The rest of the deck is comprised of the punched output from CORR.

It may be necessary at this point to make some corrections to EVAL. As in the previous program the magnitudes were presumed negative therefore if the data is positive and this includes the estimated depth of minimum, it will be necessary to convert these quantities to negative values when computing their respective luminosities by changing (0.4 * U(I)) to (-0.4 * U(I) and (0.4 * ULMP) to (-0.4*ULMP). Again as in CORR certain information was suppressed by placing a C in column 1, but a quick survey of the accompanying listing will reveal which statements should be altered. If using the program presently stored at the University of Alberta, another card containing the integer 1 in column 1 will have to be placed at the end of the input deck. If the following listing is being used as a guide no extra cards are required. If any unresolvable problems should occur, the author of these programs can be reach by addressing correspondence to:

LORENCE TOMASZEWSKI
Dept. of Astronomy
Univ. of WESTERN ONTARIO
London, Ontario

```
      REAL PHASE(500),PHANG(500),U(500),V(500),B(500),Y(500),
     1PI/3.141593/,UNORM(500),VNORM(500),BNORM(500),YNORM(500),
     1XMAT(4,500),XLAMP(10),XLAMS(10),XKP(5),XKT(5),
     1UNORMT(500),VNORMT(500),BNORMT(500),YNORMT(500),
     1T1(20),T2(20),T3(20),T4(20)
      READ(5,1) T1
    1 FORMAT(20A4)
      READ(5,2) T2
    2 FORMAT(20A4)
      READ(5,3) T3
    3 FORMAT(20A4)
      READ(5,4) T4
    4 FORMAT(20A4)
      READ(5,111) ULMP,ULMS,VLMP,VLMS,BLMP,BLMS,YLMP,VLMS
  111 FORMAT(8F8.3)
      READ(5,222) IFI1,IFI2,NEAR1,IFPS1,
     1            IFI3,IFI4,NEAR2,
     1            IFI5,IFI6,NEAR3,IFPS2,
     1            IFI7,IFI8,NEAR4
  222 FORMAT(14I4)
      READ(5,333) MAX
  333 FORMAT(I3)
      DO 101 I=1,MAX
      READ(5,555) PHASE(I),U(I),V(I),B(I),Y(I)
  555 FORMAT(F6.4,1X,F7.3,1X,F7.3,1X,F7.3,1X,F7.3)
      PHANG(I)=PHASE(I)*2.*PI
      XMAT(1,I)=10.0**(0.4*U(I))
      XMAT(2,I)=10.0**(0.4*V(I))
      XMAT(3,I)=10.0**(0.4*B(I))
      XMAT(4,I)=10.0**(0.4*Y(I))
C     WRITE(6,548) PHANG(I),XMAT(1,I),XMAT(2,I),XMAT(3,I),XMAT(4,I)
C548  FORMAT(1X,F6.4,1X,F7.3,1X,F7.3,1X,F7.3,1X,F7.3)
  101 CONTINUE
      XLAMP(1)=10.0**(0.4*ULMP)
      XLAMS(1)=10.0**(0.4*ULMS)
      XLAMP(2)=10.0**(0.4*VLMP)
```

```
XKT(JJ)=(XKT1+XKT2+XKT3+XKT4)/4.
TL1S11=XL1SN(IFI2,1,IEPS1,NEAR1,XLAMP,TS1PA,XKT,JJ,PHANG)
TL1S21=XL1SN(IFI2,2,IEPS1,NEAR1,XLAMP,TS2PA,XKT,JJ,PHANG)
TL1C11=XL1CN(IFI2,1,IEPS1,NEAR1,XLAMP,TC1PA,XKT,JJ,PHANG,2)
TL1C21=XL1CN(IFI2,2,IEPS1,NEAR1,XLAMP,TC2PA,XKT,JJ,PHANG,2)
TL1C01=XL1CN(IFI2,0,IEPS1,NEAR1,XLAMP,COT2,XKT,JJ,PHANG,2)
TL1S12=XL1SN(IFI4,1,IEPS1,NEAR2,XLAMP,TS1PD,XKT,JJ,PHANG)
TL1S22=XL1SN(IFI4,2,IEPS1,NEAR2,XLAMP,TS2PD,XKT,JJ,PHANG)
TL1C12=XL1CN(IFI4,1,IEPS1,NEAR2,XLAMP,TC1PD,XKT,JJ,PHANG,0)
TL1C22=XL1CN(IFI4,2,IEPS1,NEAR2,XLAMP,TC2PD,XKT,JJ,PHANG,0)
TL1C02=XL1CN(IFI4,0,IEPS1,NEAR2,XLAMP,COT4,XKT,JJ,PHANG,0)
TL1S13=XL1SN(IFI6,1,IEPS2,NEAR3,XLAMS,TS1SA,XKT,JJ,PHANG)
TL1S23=XL1SN(IFI6,2,IEPS2,NEAR3,XLAMS,TS2SA,XKT,JJ,PHANG)
TL1C13=XL1CN(IFI6,1,IEPS2,NEAR3,XLAMS,TC1SA,XKT,JJ,PHANG,1)
TL1C23=XL1CN(IFI6,2,IEPS2,NEAR3,XLAMS,TC2SA,XKT,JJ,PHANG,1)
TL1C03=XL1CN(IFI6,0,IEPS2,NEAR3,XLAMS,COT6,XKT,JJ,PHANG,1)
TL1S14=XL1SN(IFI8,1,IEPS2,NEAR4,XLAMS,TS1SD,XKT,JJ,PHANG)
TL1S24=XL1SN(IFI8,2,IEPS2,NEAR4,XLAMS,TS2SD,XKT,JJ,PHANG)
TL1C14=XL1CN(IFI8,1,IEPS2,NEAR4,XLAMS,TC1SD,XKT,JJ,PHANG,3)

TL1C24=XL1CN(IFI8,2,IEPS2,NEAR4,XLAMS,TC2SD,XKT,JJ,PHANG,3)
TL1C04=XL1CN(IFI8,0,IEPS2,NEAR4,XLAMS,COT8,XKT,JJ,PHANG,3)
EAPT=TL1C01/(1.-XLAMP(JJ)*XKT(JJ))
EDPT=TL1C02/(1.-XLAMP(JJ)*XKT(JJ))
EAST=TL1C03/(1.-XLAMS(JJ)*XKT(JJ))
EDST=TL1C04/(1.-XLAMS(JJ)*XKT(JJ))
F1APT=TL1S11/TL1C11
F2APT=TL1S21/TL1C21
RAPT=F1APT/F2APT
F1DPT=TL1S12/TL1C12
F2DPT=TL1S22/TL1C22
RDPT=F1DPT/F2DPT
F1AST=TL1S13/TL1C13
F2AST=TL1S23/TL1C23
RAST=F1AST/F2AST
F1DST=TL1S14/TL1C14
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```
      F2DST=TL1S24/TL1C24
      RDST=F1DST/F2DST
      WRITE(6,665) COT1,COT2,TC1PA,TC2PA,TS1PA,TS2PA,
     1             COT3,COT4,TC1PD,TC2PD,TS1PD,TS2PD,
     1             COT5,COT6,TC1SA,TC2SA,TS1SA,TS2SA,
     1             COT7,COT8,TC1SD,TC2SD,TS1SD,TS2SD
  665 FORMAT('1',22X,'CALC. OF INCOMPLETE TRANSFORMS BY THE TRAPAZOIDAL
     1RULE'/
     1'0',5X,'MINIMA',15X,'CO(1)',8X,'CO(2)',9X,'C1',11X,'C2',11X,'S1',
     111X,'S2'///2X,'PRIMARY MIN.'//7X,'A',16X,F8.6,5X,F8.6,5X,F8.6,5X,
     1F8.6,5X,F8.6,5X,F8.6//7X,'D',16X,F8.6,5X,F8.6,5X,F8.6,5X,F8.6,5X,
     1F8.6,5X,F8.6///2X,'SEC. MIN.'//7X,'A',16X,F8.6,5X,F8.6,5X,F8.6,
     15X,F8.6,5X,F8.6,5X,F8.6//7X,'D',16X,F8.6,5X,F8.6,5X,F8.6,5X,F8.6,
     15X,F8.6,5X,F8.6)
      WRITE(6,776) XKT(JJ)
  776 FORMAT('-THE NORMALIZATION FACTOR IS  ',F8.6)
      WRITE(6,887) XKT1,TL1C01,F1APT,F2APT,RAPT,EAPT,
     1             XKT2,TL1C02,F1DPT,F2DPT,RDPT,EDPT,
     1             XKT3,TL1C03,F1AST,F2AST,RAST,EAST,
     1             XKT4,TL1C04,F1DST,F2DST,RDST,EDST
  887 FORMAT('-',22X,'CALC. OF CHAR. FCTS E,F1,F2'/'0',5X,'MINIMA',16X,
     1'K',9X,'D=I1C0',9X,'F1',11X,'F2',10X,'F1/F2',9X,'F'///
     12X,'PRIMARY MIN.'//7X,'A',16X,F8.6,5X,F8.6,5X,F8.6,5X,F8.6,5X,
     1F8.6,5X,F8.6//7X,'D',16X,F8.6,5X,F8.6,5X,F8.6,5X,F8.6,5X,F8.6,
     15X,F8.6///2X,'SEC. MIN.'//7X,'A',16X,F8.6,5X,F8.6,5X,F8.6,5X,
     1F8.6,5X,F8.6,5X,F8.6//7X,'D',16X,F8.6,5X,F8.6,5X,F8.6,5X,F8.6,
     15X,F8.6,5X,F8.6)
  212 CONTINUE
      UPT=XLAMP(1)*XKT(1)
      UST=XLAMS(1)*XKT(1)
      VPT=XLAMP(2)*XKT(2)
      VST=XLAMS(2)*XKT(2)
      BPT=XLAMP(3)*XKT(3)
      BST=XLAMS(3)*XKT(3)
      YPT=XLAMP(4)*XKT(4)
      YST=XLAMS(4)*XKT(4)
```

```
XLAMS(2)=10.0**(0.4*VLMS)
XLAMP(3)=10.0**(0.4*BLMP)
XLAMS(3)=10.0**(0.4*BLMS)
XLAMP(4)=10.0**(0.4*YLMP)
XLAMS(4)=10.0**(0.4*YLMS)
DO  212 K=1,4
JJ=K
COT1=CNT(XMAT,PHANG,0,NEAR1,IFI1,IEPS1,JJ,XLAMP,PHASE,2)
COT2=CNT(XMAT,PHANG,0,NEAR1,IFI2,IEPS1,JJ,XLAMP,PHASE,2)
XKT1=(PHANG(IFI1)-PHANG(IFI2))/(COT1-COT2)
TS1PA=SNT(XMAT,PHANG,1,NEAR1,IFI2,IEPS1,JJ,XLAMP,PHASE,2)
TS2PA=SNT(XMAT,PHANG,2,NEAR1,IFI2,IEPS1,JJ,XLAMP,PHASE,2)
TC1PA=CNT(XMAT,PHANG,1,NEAR1,IFI2,IEPS1,JJ,XLAMP,PHASE,2)
TC2PA=CNT(XMAT,PHANG,2,NEAR1,IFI2,IEPS1,JJ,XLAMP,PHASE,2)
COT3=CNT(XMAT,PHANG,0,NEAR2,IFI3,IEPS1,JJ,XLAMP,PHASE,0)
COT4=CNT(XMAT,PHANG,0,NEAR2,IFI4,IEPS1,JJ,XLAMP,PHASE,0)
XKT2=(PHANG(IFI4)-PHANG(IFI3))/(COT3-COT4)
TS1PD=SNT(XMAT,PHANG,1,NEAR2,IFI4,IEPS1,JJ,XLAMP,PHASE,0)
TS2PD=SNT(XMAT,PHANG,2,NEAR2,IFI4,IEPS1,JJ,XLAMP,PHASE,0)
TC1PD=CNT(XMAT,PHANG,1,NEAR2,IFI4,IEPS1,JJ,XLAMP,PHASE,0)
TC2PD=CNT(XMAT,PHANG,2,NEAR2,IFI4,IEPS1,JJ,XLAMP,PHASE,0)
COT5=CNT(XMAT,PHANG,0,NEAR3,IFI5,IEPS2,JJ,XLAMS,PHASE,1)
COT6=CNT(XMAT,PHANG,0,NEAR3,IFI6,IEPS2,JJ,XLAMS,PHASE,1)
XKT3=(PHANG(IFI5)-PHANG(IFI6))/(COT5-COT6)
TS1SA=SNT(XMAT,PHANG,1,NEAR3,IFI6,IEPS2,JJ,XLAMS,PHASE,1)
TS2SA=SNT(XMAT,PHANG,2,NEAR3,IFI6,IEPS2,JJ,XLAMS,PHASE,1)
TC1SA=CNT(XMAT,PHANG,1,NEAR3,IFI6,IEPS2,JJ,XLAMS,PHASE,1)
TC2SA=CNT(XMAT,PHANG,2,NEAR3,IFI6,IEPS2,JJ,XLAMS,PHASE,1)
COT7=CNT(XMAT,PHANG,0,NEAR4,IFI7,IEPS2,JJ,XLAMS,PHASE,1)
COT8=CNT(XMAT,PHANG,0,NEAR4,IFI8,IEPS2,JJ,XLAMS,PHASE,1)
XKT4=(PHANG(IFI8)-PHANG(IFI7))/(COT7-COT8)
TS1SD=SNT(XMAT,PHANG,1,NEAR4,IFI8,IEPS2,JJ,XLAMS,PHASE,1)
TS2SD=SNT(XMAT,PHANG,2,NEAR4,IFI8,IEPS2,JJ,XLAMS,PHASE,1)
TC1SD=CNT(XMAT,PHANG,1,NEAR4,IFI8,IEPS2,JJ,XLAMS,PHASE,1)
TC2SD=CNT(XMAT,PHANG,2,NEAR4,IFI8,IEPS2,JJ,XLAMS,PHASE,1)
```

```
      WRITE(6,175) UPT,UST,VPT,VST,BPT,BST,YPT,YST
  175 FORMAT('1',20X,'DEPTH OF MIN.'///20X,'PRIM.',4X,'SEC.'///
     15X,'U',14X,F4.3,5X,F4.3//5X,'V',14X,F4.3,5X,F4.3//5X,'B',14X,F4.3,

     15X,F4.3//5X,'Y',14X,F4.3,5X,F4.3)
      WRITE(7,176) UPT,UST,VPT,VST,BPT,BST,YPT,YST
  176 FORMAT(F4.3,1X,F4.3,1X,F4.3,1X,F4.3,1X,F4.3,1X,F4.3,
     11X,F4.3,1X,F4.3)
      DO 414 I=1,MAX
      UNORMT(I)=XMAT(1,I)*XKT(1)
      VNORMT(I)=XMAT(2,I)*XKT(2)
      BNORMT(I)=XMAT(3,I)*XKT(3)
      YNORMT(I)=XMAT(4,I)*XKT(4)
      WRITE(6,112) PHASE(I),XMAT(1,I),UNORMT(I),XMAT(2,I),VNORMT(I),
     1             XMAT(3,I),BNORMT(I),XMAT(4,I),YNORMT(I)
  122 FORMAT(F6.4,8F9.4)
      WRITE(7,133) PHASE(I), UNORMT(I),VNORMT(I),BNORMT(I),YNORMT(I)
  133 FORMAT(F6.4,1X,F7.3,1X,F7.3,1X,F7.3,1X,F7.3)
  414 CONTINUE
C     CALL CGPL(PHASE,UNORMT,UNORMT,MAX,1,1,1,1,1,-.05,0.2,5.5,
C    10.0,0.2,6.0,T1,6)
C     CALL CGPL(PHASE,VNORMT,VNORMT,MAX,1,1,1,1,1,-.05,0.2,5.5,
C    10.0,0.2,6.0,T2,6)
C     CALL CGPL(PHASE,BNORMT,BNORMT,MAX,1,1,1,1,1,-.05,0.2,5.5,
C    10.0,0.2,6.0,T3,6)
C     CALL CGPL(PHASE,YNORMT,YNORMT,MAX,1,1,1,1,1,-.05,0.2,5.5,
C    10.0,0.2,6.0,T4,6)
C     CALL CGPL(PHASE,YNORMT,YNORMT,MAX,0,1,1,1,1,-.05,0.2,5.5,
C    10.0,0.2,6.0,T4,6)
      STOP
C     DEBUG INIT(COT1,COT2,XKT1,TS1PA,TC1PA,TC2PA,TS2PA,COT3,COT4,
C    1XKT2,TS1PD,TS2PD,TC1PD,TC2PD,COT5,COT6,XKT3,TS1SA,TS2SA,TC1SA,
C    1TC2SA,COT7,COT8,XKT4,TS1SD,TS2SD,TC1SD,TC2SD,FAPT,FDPT,FAST,FDST,
C    1TL1C11,TL1C21,TL1C12,TL1C22,F1APT,F2APT,FAPT,F1DPT,F2DPT,FDPT,F1AS
C    1,F2AST,FAST,F1DST,F2DST,FDST)
      END
```

```
      FUNCTION SNT(XMAT,PHANG,N,NEAR,IFI,IEPS,JJ,XLAM,PHASE,IBRAN)
      REAL XMAT(4,500),PHANG(500),PHASE(500),XLAM(4),ANGLE(500)
      IMP=IBRAN
      IT=0
      IF(IBRAN.EQ.0) IT=2
      SUM=0.0
      DIFF=0.0
      IF(IBRAN.EQ.1) IMP=IBRAN-1
      IF(NEAR.GT.IFI) GO TO 110
      IF((IEPS-NEAR).EQ.0) GO TO 220
      PEPSI=PHANG(IEPS)
      SHIFT=PHANG(NEAR)+(IMP*3.141593)
      CORREC=(XMAT(JJ,IEPS)*ABS(SIN(N*PEPSI))+XMAT(JJ,NEAR)*ABS(SIN
     1(N*SHIFT)))*(SHIFT-PEPSI)
      GO TO 193
  220 CORREC=0.0
  193 J=NEAR
      K=IFI-1
      DO 101 I=J,IFI
      ANGLE(I)=PHANG(I)+(IMP*3.141593)
  101 CONTINUE
      DO 102 I=J,K
      SN=(XMAT(JJ,I)*ABS(SIN(N*ANGLE(I)))+XMAT(JJ,I+1)*ABS(SIN(
     1N*ANGLE(I+1))))*(ANGLE(I+1)-ANGLE(I))
C     WRITE(6,839) XMAT(JJ,I),XMAT(JJ,I+1),ANGLE(I+1),ANGLE(I)
C839  FORMAT(1X,F8.3,F8.3,F8.4,F8.4)
      DIFF=DIFF+(PHASE(I+1)-PHASE(I))
      SUM=SUM+SN
  102 CONTINUE
      H=DIFF/(IFI-NEAR)
      SNTT=0.5*(SUM+CORREC)-(H**2*N*(XMAT(JJ,IFI)*COS(N*PHANG(IFI))-
     1XMAT(JJ,NEAR)))/12.0
      SNT=ABS(SNTT)
      RETURN
```

```
 110  J=IFI
      K=NEAR-1
      IF((IEPS-NEAR).EQ.0) GO TO 181
      PEPSI=PHANG(IEPS)+((IT+IMP)*3.141593)
      SHIFI=PHANG(NEAR)
      CORREC=(XMAT(JJ,IEPS)*ABS(SIN(N*PEPSI))+XMAT(JJ,NEAR)*ABS(SIN(N*
     1SHIFT)))*(PEPSI-SHIFT)
      GO TO 184
 181  CORREC=0.0
 184  DO 221 I=J,NEAR
      ANGLE(I)=PHANG(I)+(IMP*3.141593)
 221  CONTINUE
      DO 222 I=J,K
      SN=(XMAT(JJ,I)*ABS(SIN(N*ANGLE(I)))+XMAT(JJ,I+1)*
     1ABS(SIN(N*ANGLE(I+1))))*(ANGLE(I+1)-ANGLE(I))
C     WRITE(6,840) XMAT(JJ,I),XMAT(JJ,I+1),ANGLE(I+1),ANGLE(I)
C840  FORMAT(1X,F8.3,F8.3,F8.4,F8.4)
      DIFF=DIFF+(PHASE(I+1)-PHASE(I))
      SUM=SUM+SN
 222  CONTINUE
      H=DIFF/(NEAR-IFI)

      SNTT=0.5*(SUM+CORREC)-(H**2*N*(XMAT(JJ,IFI)*COS(N*PHANG(IFI)
     1)-XMAT(JJ,NEAR)))/12.0
      SNT=ABS(SNTT)
      RETURN
C     DEBUG INIT(NEAR,N,IFI,IEPS,PEPSI,SHIFT,SNT,IMP)
      END
```

```
      FUNCTION CNI(XMAT,PHANG,N,NEAR,IFI,IEPS,JJ,XLAM,PHASE,IBRAN)
      REAL XMAT(4,500),PHANG(500),PHASE(500),XLAM(4),ANGLE(500)
      IMP=IBRAN
      IT=0
      IF(IBRAN.EQ.0) IT=2
      IF(IBRAN.EQ.1) IMP=IBRAN-1
      SUM=0.0
      DIFF=0.0
      IF(NEAR.GT.IFI) GO TO 110
      IF((IEPS-NEAR).EQ.0) GO TO 220
      PEPSI=PHANG(IEPS)
      SHIFT=PHANG(NEAR)+(IMP*3.141593)
      CORREC=(XMAT(JJ,IEPS)*ABS(COS(N*PEPSI))+XMAT(JJ,NEAR)*ABS(COS
     1(N*SHIFT)))*(SHIFT-PEPSI)
      GO TO 193
 220  CORREC=0.0
 193  J=NEAR
      K=IFI-1
      DO 101 I=J,IFI
      ANGLE(I)=PHANG(I)+(IMP*3.141593)
 101  CONTINUE
      DO 102 I=J,K
      CN=(XMAT(JJ,I)*ABS(COS(N*ANGLE(I)))+XMAT(JJ,I+1)*ABS(COS
     1(N*ANGLE(I+1))))*(ANGLE(I+1)-ANGLE(I))
C     WRITE(6,837) XMAT(JJ,I),XMAT(JJ,I+1),ANGLE(I+1),ANGLE(I)
C837  FORMAT(1X,F8.3,F8.3,F8.4,F8.4)
      DIFF=DIFF+(PHASE(I+1)-PHASE(I))
      SUM=SUM+CN
 102  CONTINUE
      H=DIFF/(IFI-NEAR)
      CNTI=0.5*(SUM+CORREC)+((H**2*N*XMAT(JJ,IFI)
     1*SIN(N*PHANG(IFI)))/12.0)
      CNI=ABS(CNTI)
      RETURN
```

```
 110  J=IFI
      K=NEAR-1
      IF((IEPS-NEAR).EQ.0) GO TO 181
      PEPSI=PHANG(IEPS)+((IMP+IT)*3.141593)
      SHIFT=IHANG(NEAR)
      CORREC=(XMAT(JJ,IEPS)*ABS(COS(N*PEPSI))+XMAT(JJ,NEAR)
     1*ABS(COS(N*SHIFT)))*(PEPSI-SHIFT)
      GO TO 184
 181  CORREC=0.0
 184  DO 221 I=J,NEAR
      ANGLE(I)=PHANG(I)+(IMP*3.141593)
 221  CONTINUE
      DO 222 I=J,K
      CN=(XMAT(JJ,I)*ABS(COS(N*ANGLE(I)))+XMAT(JJ,I+1)
     1*ABS(COS(N*ANGLE(I+1))))*(ANGLE(I+1)-ANGLE(I))
C     WRITE(6,838) XMAT(JJ,I),XMAT(JJ,I+1),ANGLE(I+1),ANGLE(I)
C838  FORMAT(1X,F8.3,F8.3,F8.4,F8.4)
      DIFF=DIFF+(PHASE(I+1)-PHASE(I))
      SUM=SUM+CN
 222  CONTINUE
      H=DIFF/(NEAR-IFI)

      CNTT=0.5*(SUM+CORREC)+((H**2*N*XMAT(JJ,IFI)
     1*SIN(N*PHANG(IFI)))/12.0)
      CNT=ABS(CNTT)
      RETURN
C     DEBUG INIT(NEAR,IFI,IEPS,N,JJ,CORREC,SHIFT,PEPSI,H,
C    1CNT,J,K,IMP)
      END
```

```
      FUNCTION XL1CN(IFI,N,IEPS,NEAR,XLAM,CN,XK,JJ,PHANG,IBRAN)
      REAL PHANG(500),XK(4),XLAM(4)
      IF(N.EQ.0) GO TO 110
      IF((NEAR-IEPS).EQ.0) GO TO 100
      PEPSI=-1.*ABS(SIN(N*PHANG(IEPS)))
      GO TO 200
100   PEPSI=ABS(SIN(N*PHANG(IEPS)))
200   DUMMY=0.
      XL1CN=(ABS(SIN(N*PHANG(IFI)))/N)-((PEPSI/N)
     1*XLAM(JJ)*XK(JJ))-(XK(JJ)*CN)
      RETURN
110   IF(IBRAN.EQ.2) GO TO 222
      IF(IBRAN.EQ.0) GO TO 111
      IF(IBRAN.EQ.1) GO TO 444
      IF((IEPS-NEAR).EQ.0) GO TO 331
      X=3.141593-PHANG(IFI)
      Y=3.141593-PHANG(IEPS)
      GO TO 121
331   X=3.141593-PHANG(IFI)
      Y=3.141593-PHANG(IEPS)
      GO TO 121
222   D1=0.
      IF((IEPS-NEAR).EQ.0) GO TO 221
      X=PHANG(IFI)
      Y=PHANG(IEPS)-2.*3.141593
      GO TO 121
221   X=PHANG(IFI)
      Y=PHANG(IEPS)
      GO TO 121
111   D2=0.
      IF((IEPS-NEAR).EQ.0) GO TO 141
      X=2.*3.141593-PHANG(IFI)
      Y=-PHANG(IEPS)
      GO TO 121
```

```
  141 X=2.*3.141593-PHANG(IFI)
      Y=2.*3.141593-PHANG(IEPS)
      GO TO 121
  444 IF((IEPS-NEAR).NE.0) GO TO 414
      X=PHANG(IFI)-3.141593
      Y=PHANG(IEPS)-3.141593
      GO TO 121
  414 X=PHANG(IFI)-3.141593
      Y=PHANG(IEPS)-3.141593
  121 XL1CN=X-Y*XLAM(JJ)*XK(JJ)-XK(JJ)*CN
      RETURN
C     DEBUG INIT(N,XLAM,XK,PEPSI,X,Y,XL1CN)
      END
```

```
      FUNCTION XL1SN(IFI,N,IEPS,NEAR,XLAM,SN,XK,JJ,PHANG)
      REAL PHANG(500),XK(4),XLAM(4)
      IF((IEPS-NEAR).EQ.0) GO TO 100
      SIGN=-1.0
      GO TO 200
  100 SIGN=1.0
  200 XL1SN=((1.-ABS(COS(N*PHANG(IFI))))/N)-(SIGN*((1.-ABS(COS(N*
     1PHANG(IEPS))))/N)*XLAM(JJ)*XK(JJ))-(XK(JJ)*SN)
      RETURN
C     DEBUG INIT(IFI,N,IEPS,NEAR,XLAM,SN,XK,JJ,XL1SN)
      END
```

B30100